Post-Quantum Crypto is Coming!

JP Aumasson

STHACK 2023

Background

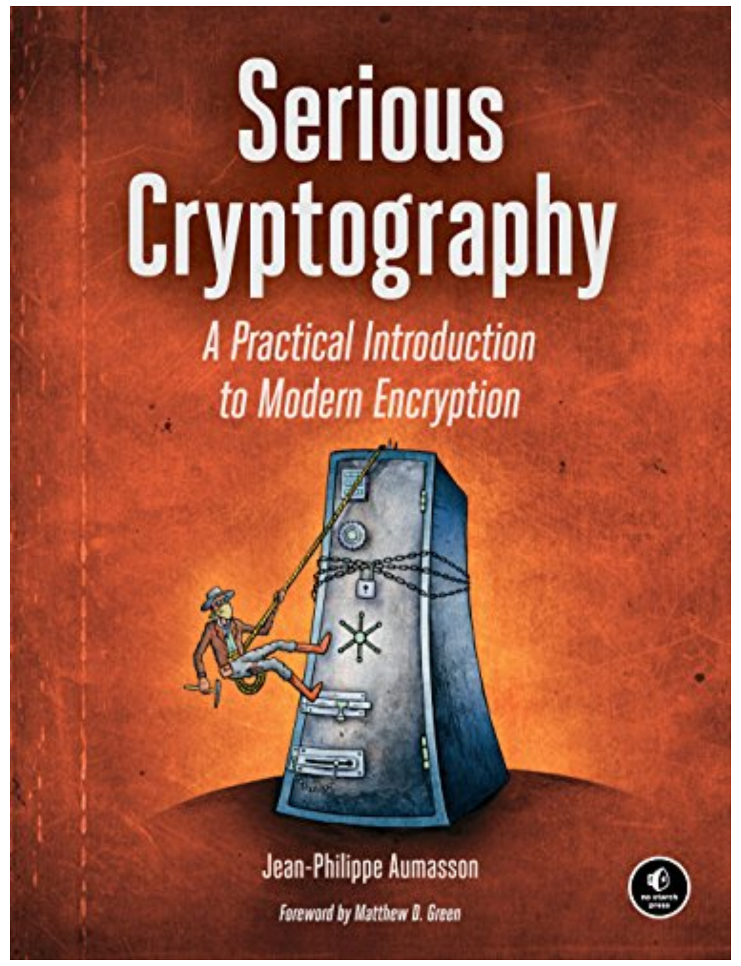
Co-founder & chief security officer of Taurus SA

- Swiss firm founded in 2018, team of 60+
- Digital asset custody tech and infrastructure, FINMA-regulated
- Working with cool tech: HSM, MPC, ZK proofs, etc.

https://taurushq.com   https://t-dx.com

- 15 years in applied crypto & security
- BLAKE2, BLAKE3, SipHash, etc.
- Some crypto*graphy* books

https://aumasson.jp



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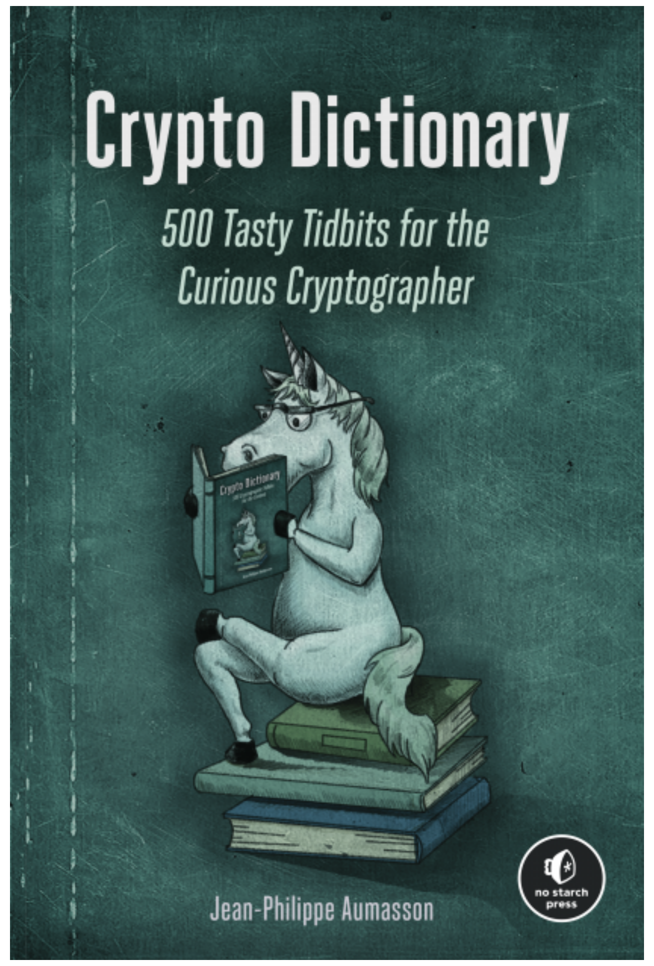

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Prerequisites

Fundamental Equations

Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$$

Time independent Schrödinger equation:

$$H\psi = E\psi, \qquad \Psi = \psi e^{-iEt/\hbar}$$

Standard Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V$$

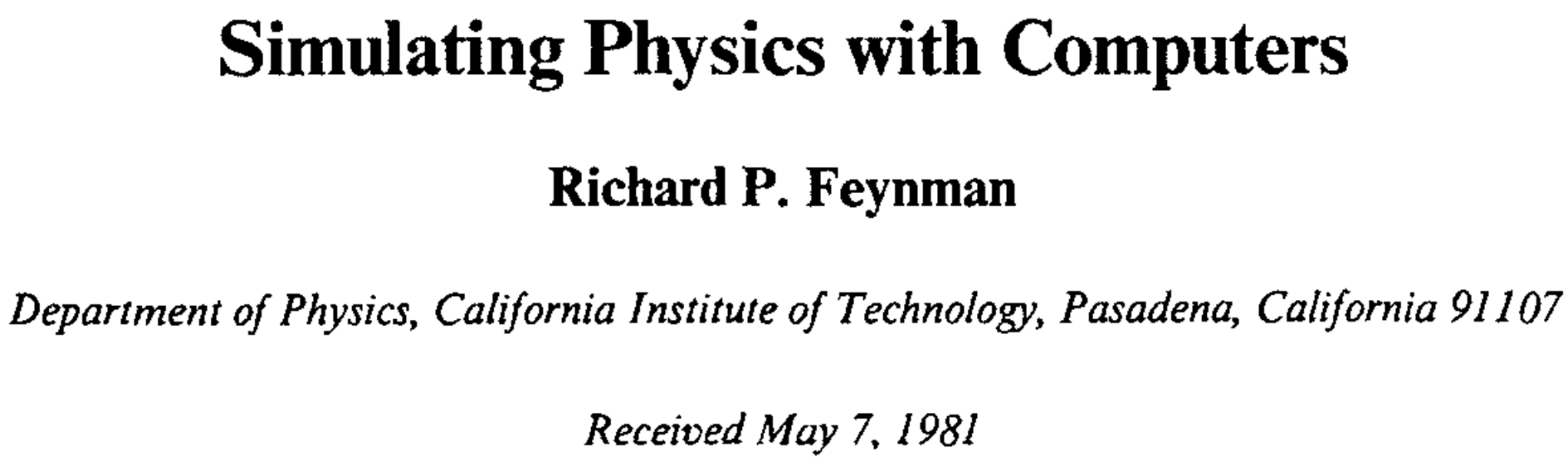
Time dependence of an expectation value:

$$\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle[H,Q]\rangle + \left\langle\frac{\partial Q}{\partial t}\right\rangle$$

Generalized uncertainty principle:

$$\sigma_A\sigma_B \geq \left|\frac{1}{2i}\langle[A,B]\rangle\right|^2$$

Why Quantum Computers?


Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

Not to Break Crypto..

5. CAN QUANTUM SYSTEMS BE PROBABILISTICALLY SIMULATED BY A CLASSICAL COMPUTER?

Now the next question that I would like to bring up is, of course, the interesting one, i.e., Can a quantum system be probabilistically simulated by a classical (probabilistic, I'd assume) universal computer? In other words, a computer which will give the same probabilities as the quantum system does. If you take the computer to be the classical kind I've described so far, (not the quantum kind described in the last section) and there're no changes in any laws, and there's no hocus-pocus, the answer is certainly, No! This is called the hidden-variable problem: it is impossible to represent the results of quantum mechanics with a classical universal device. To learn a little bit about it, I say let us try to put the quantum equations in a form as close as

But (Initially) to Simulate Quantum Physics

4. QUANTUM COMPUTERS—UNIVERSAL QUANTUM SIMULATORS

The first branch, one you might call a side-remark, is, Can you do it with a new kind of computer—a quantum computer? (I'll come back to the other branch in a moment.) Now it turns out, as far as I can tell, that you can simulate this with a quantum system, with quantum computer elements. It's not a Turing machine, but a machine of a different kind. If we disregard the continuity of space and make it discrete, and so on, as an approximation (the same way as we allowed ourselves in the classical case), it does seem to

Qubits Instead of Bits

$\alpha\,|0\rangle + \beta\,|1\rangle$ — Measure → 0 with probability $|\alpha|^2$ / 1 with probability $|\beta|^2$

Qubit state

Qubit stays 0 or 1 forever

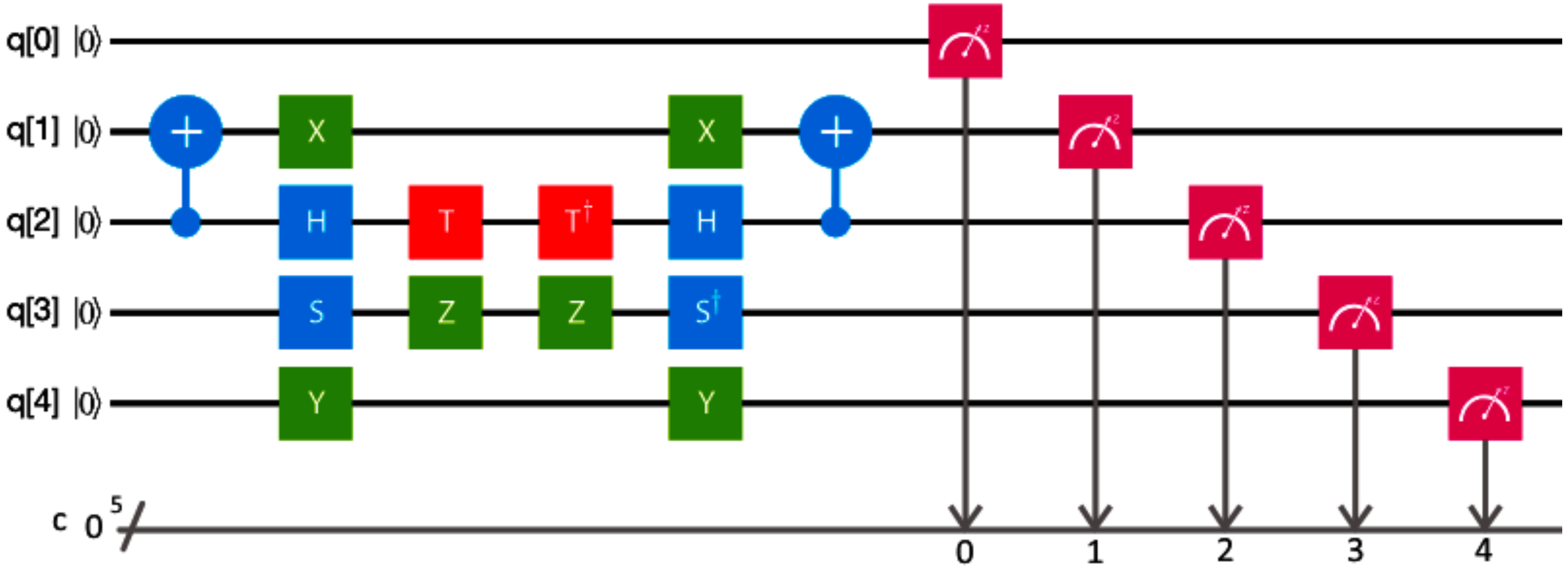
Generalizes to more than 2 states: qutrits, qubytes, etc.

α, β are complex, negative "probabilities" called **amplitudes**

Real randomness!

How Quantum Algorithms Work

Circuit of quantum gates, transforming a quantum state, ending with a measurement



Can be simulated with high-school linear algebra, but does no scale!

- **Quantum state** = vector of 2^N amplitudes for N qubits
- **Quantum gates** = matrix multiplications, with $O(2^{3N})$ complexity

Quantum Speedup

When quantum computers can solve a problem faster than classical computers

Most interesting: **Superpolynomial** quantum speedup (“exponential" boost)

List of problems on the Quantum Zoo: http://math.nist.gov/quantum/zoo/

Quantum Parallelism

Quantum computers “work" on all values simultaneously, via **superposition**

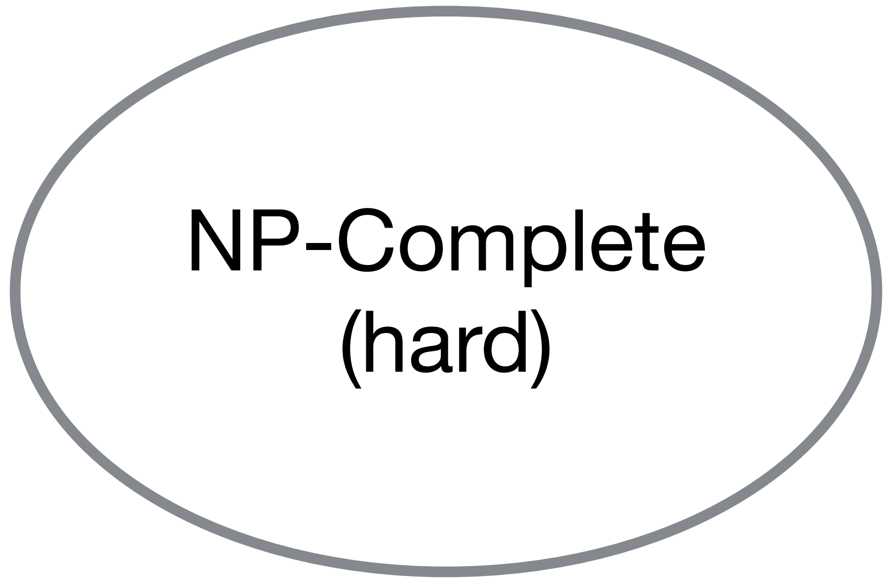
But they cannot *“try every answer in parallel and pick the best”*

You can only **observe one “value”** that results from the interference of all, through a projection from the Hilbert space (where qubits “live") to some basis

NP-complete Problems

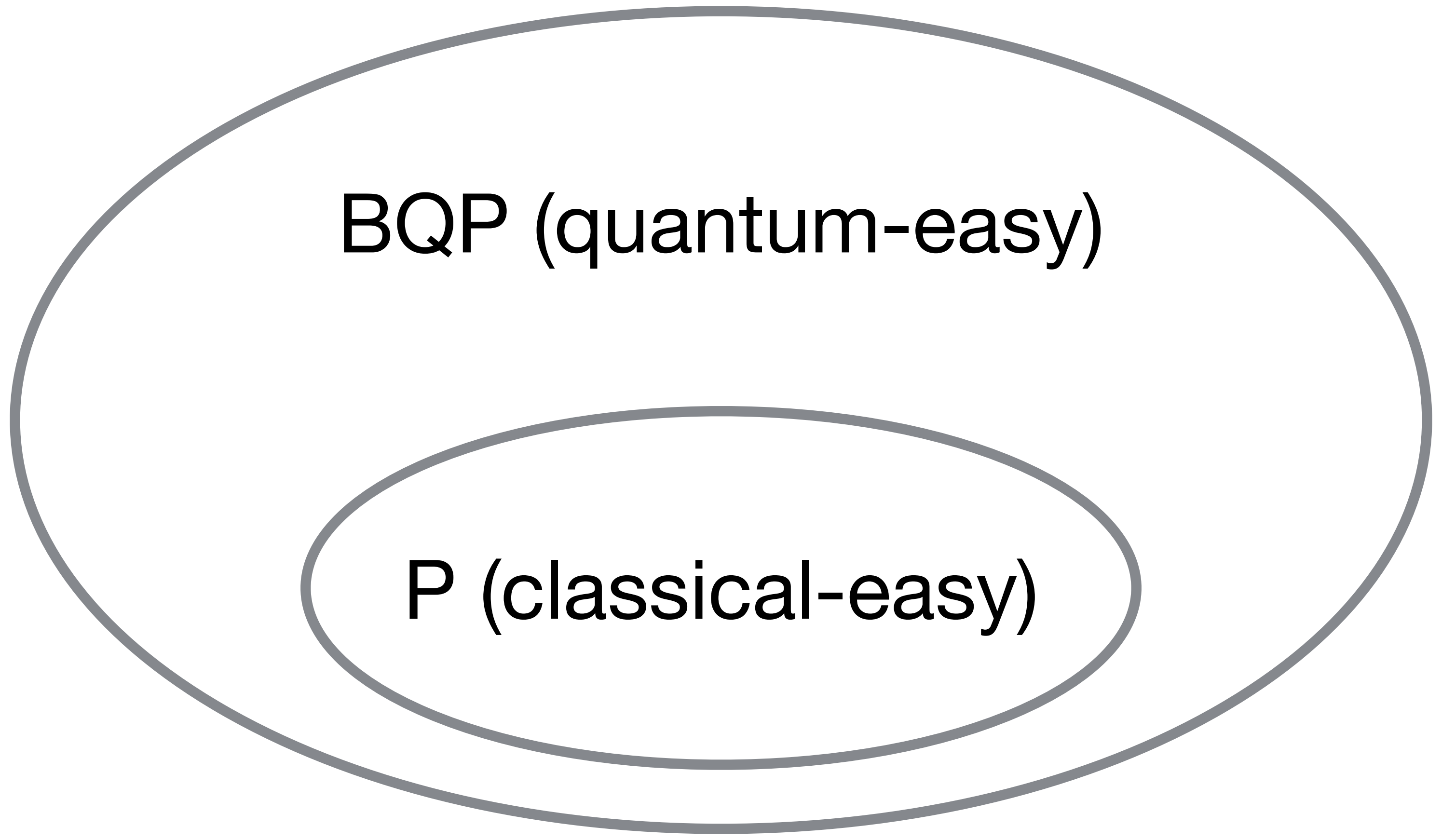
- Solution hard to find, but easy to verify
- Constraint satisfaction problems (SAT, TSP, knapsacks, etc.)
- Sometimes used in crypto (lattice problems in post-quantum schemes)

Can't be solved faster with quantum computers!



NP is not included in BQP

Therefore quantum computers can't solve NP-hard problems



BQP = bounded-error quantum polynomial time, what QC can solve efficiently

Quantum Supremacy?

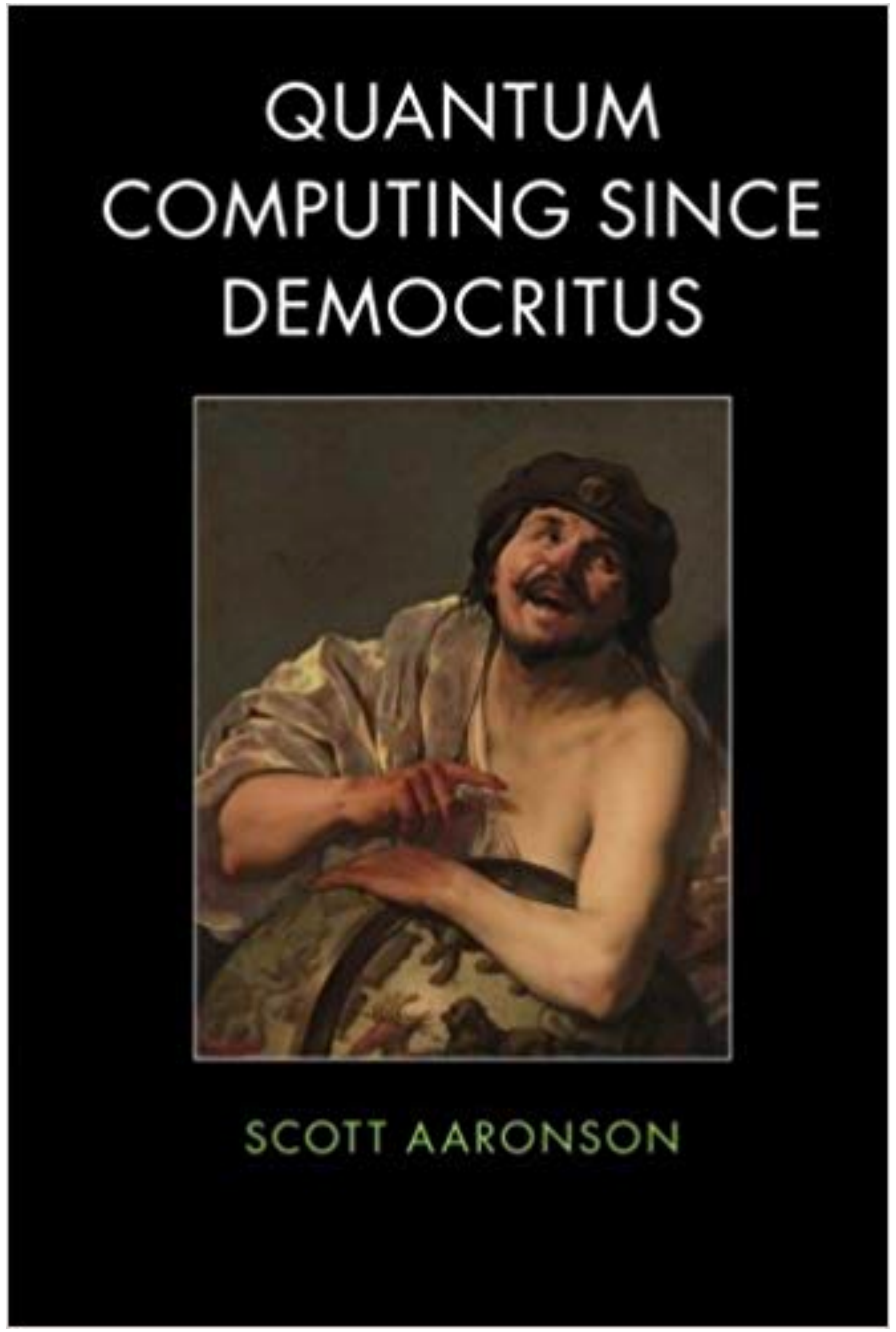
Google thinks it's close to "quantum supremacy." Here's what that really means.

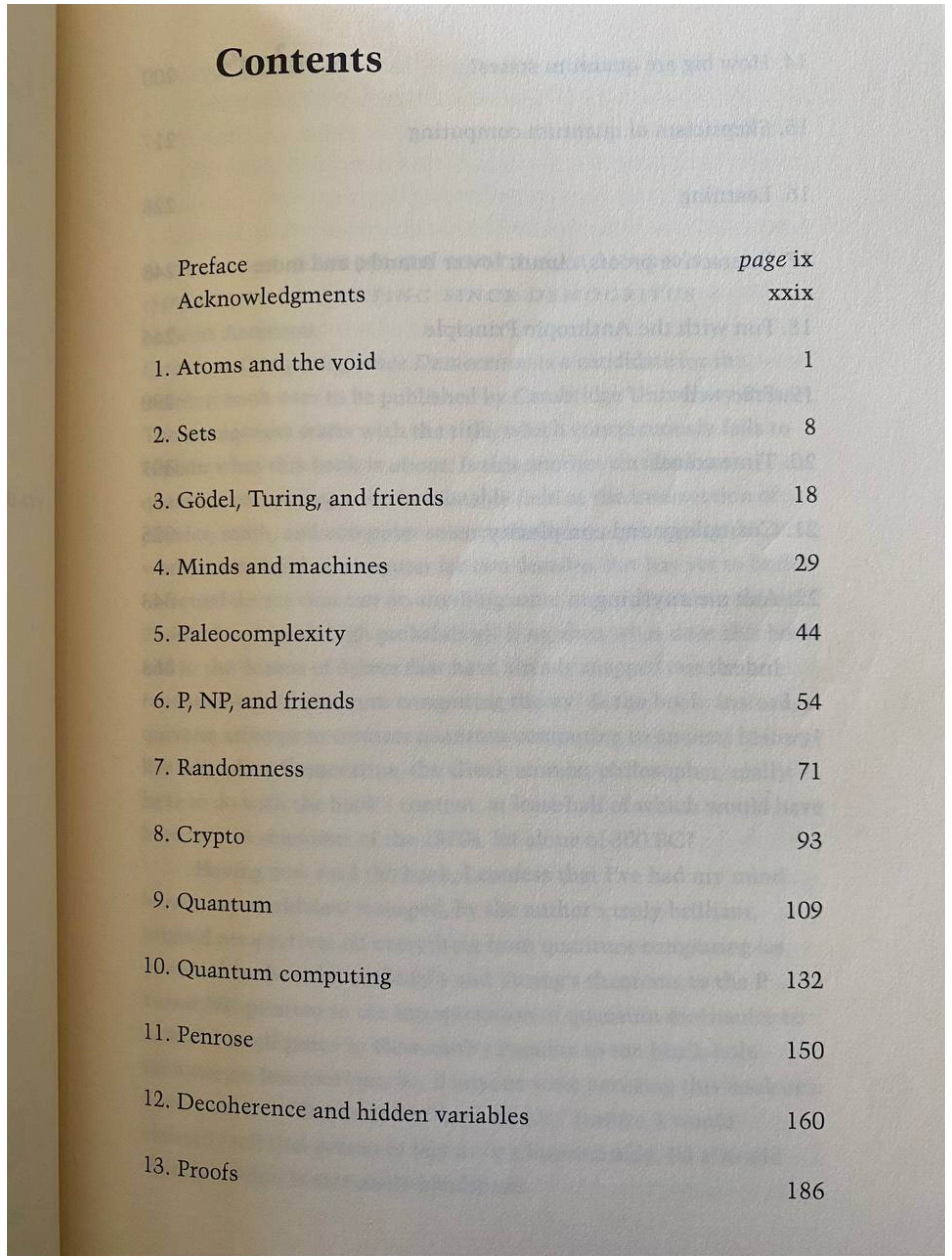
It's not the number of qubits; it's what you do with them that counts.

by Martin Giles and Will Knight March 9, 2018

Seventy-two may not be a large number, but in quantum computing terms, it's massive. This week Google unveiled Bristlecone, a new quantum computing chip with 72 quantum bits, or qubits—the fundamental units of computation

Recommended Reading

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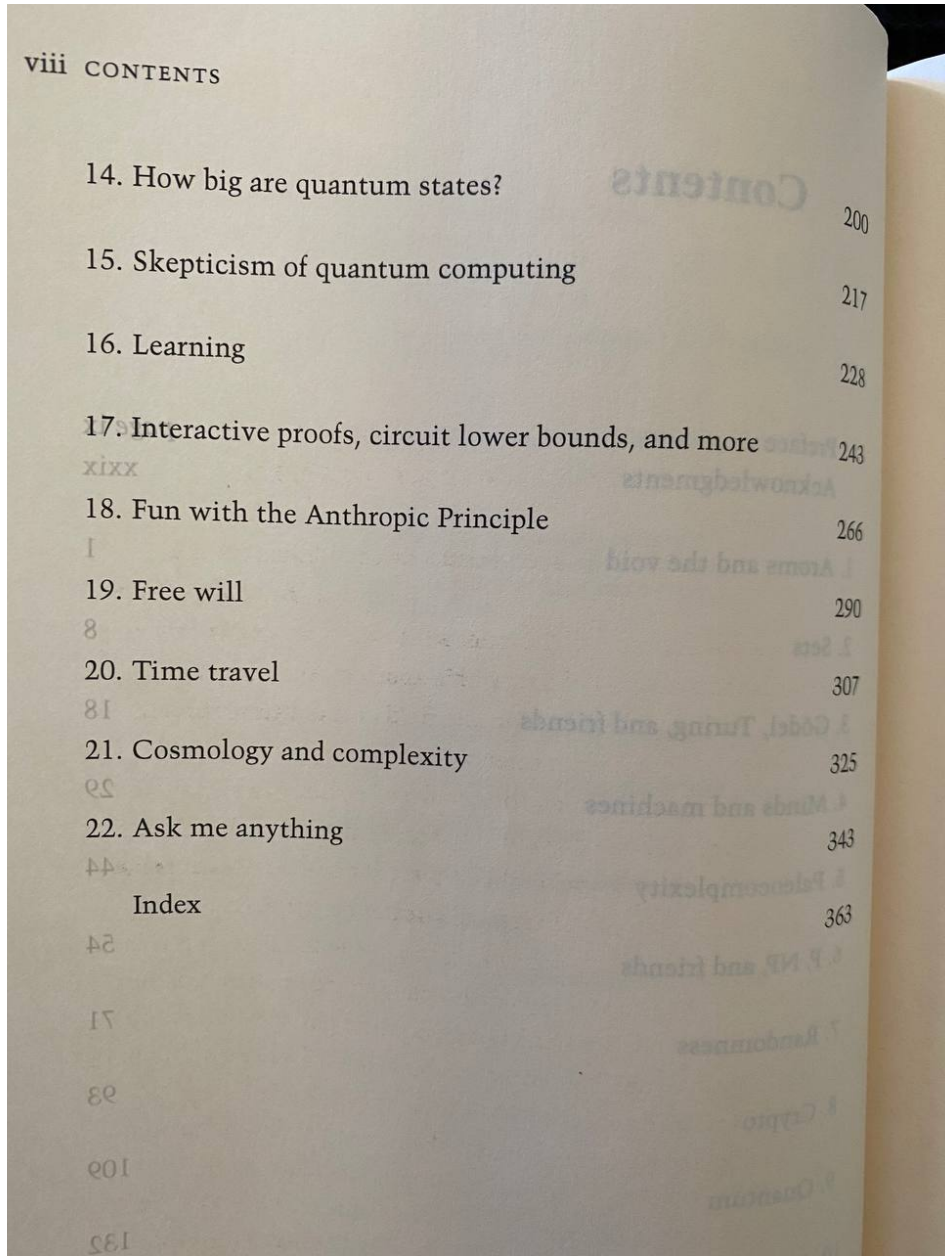
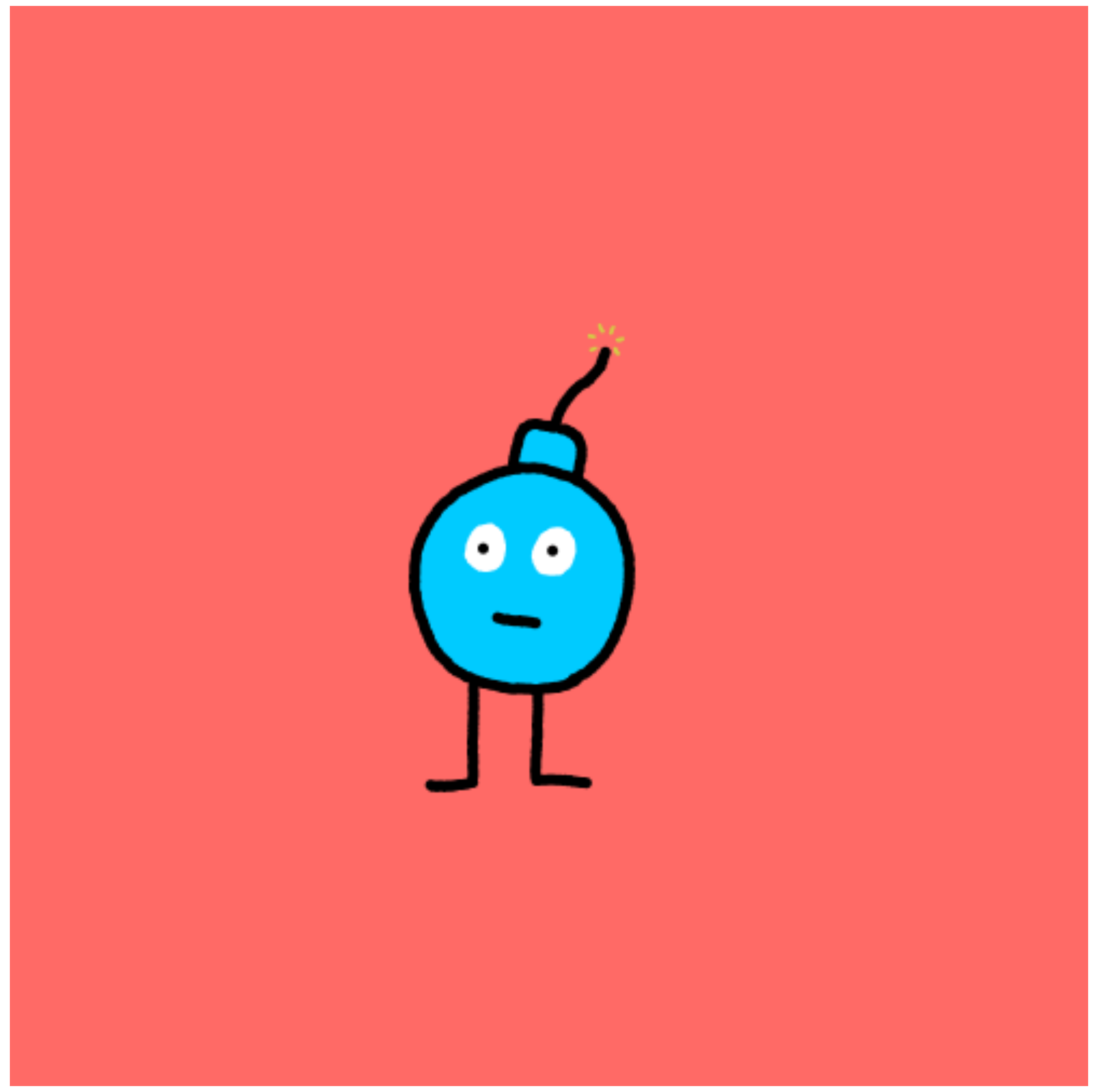


Impact on Cryptography

Shor’s Quantum Algorithm

Polynomial-time algorithm for the following problems:

- Computes **p** given **n** = **pq** → RSA dead
- Computes **d** given **y** = $\mathbf{x}^{\mathbf{d}}$ mod **p** → ECC/DH dead

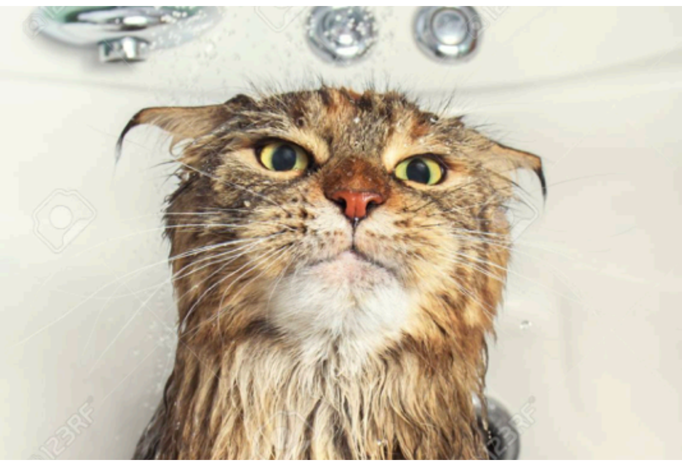


Practically impossible on a classical machine

#QuantumSpeedup

How Bad for Crypto Applications?

More bad



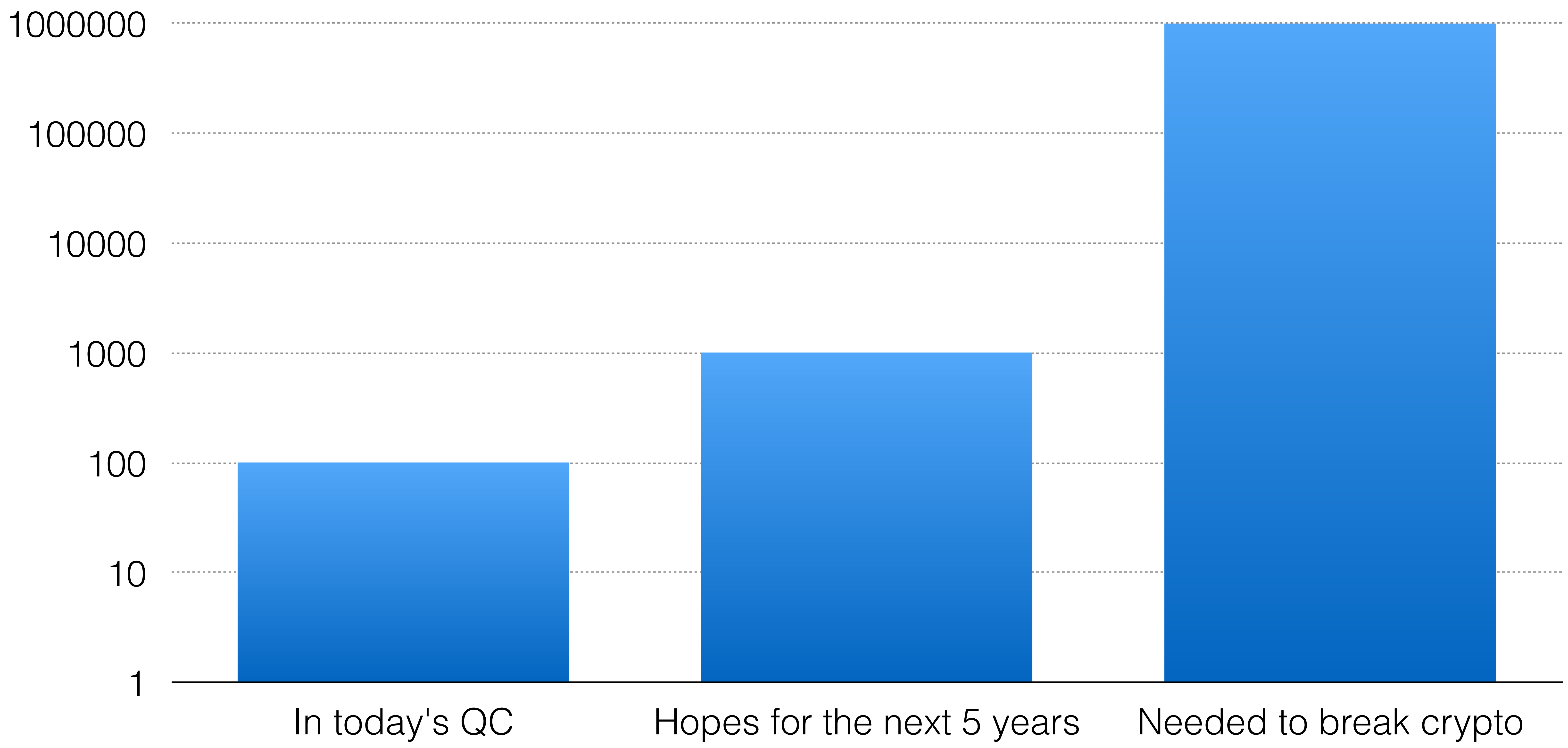
Mildly unpleasant: Signatures (ECDSA, Ed25519, etc.)
Can be reissued with a post-quantum algorithm
Use cases: Blockchains, firmware signing, application signing

Somewhat off-putting: Key agreement (DH, ECDH, KEMs, etc.) **in ratcheted** protocols: Signal's, other X3HD + Double Ratchet
Use cases: End-to-end messaging and (group) calls

Quite annoying: Key agreement (DH, ECDH, KEMs, etc.) **in single-handshake** protocols: IPsec, SSH, TLS, WireGuard
Use cases: HTTPS requests, VPNs, StartTLS, etc.

Extremely irritating: Encryption (RSA encryption, ECIES, etc.)
Encrypted messages compromised forever
Use cases: PGP email, encrypted backups

How Many Qubits

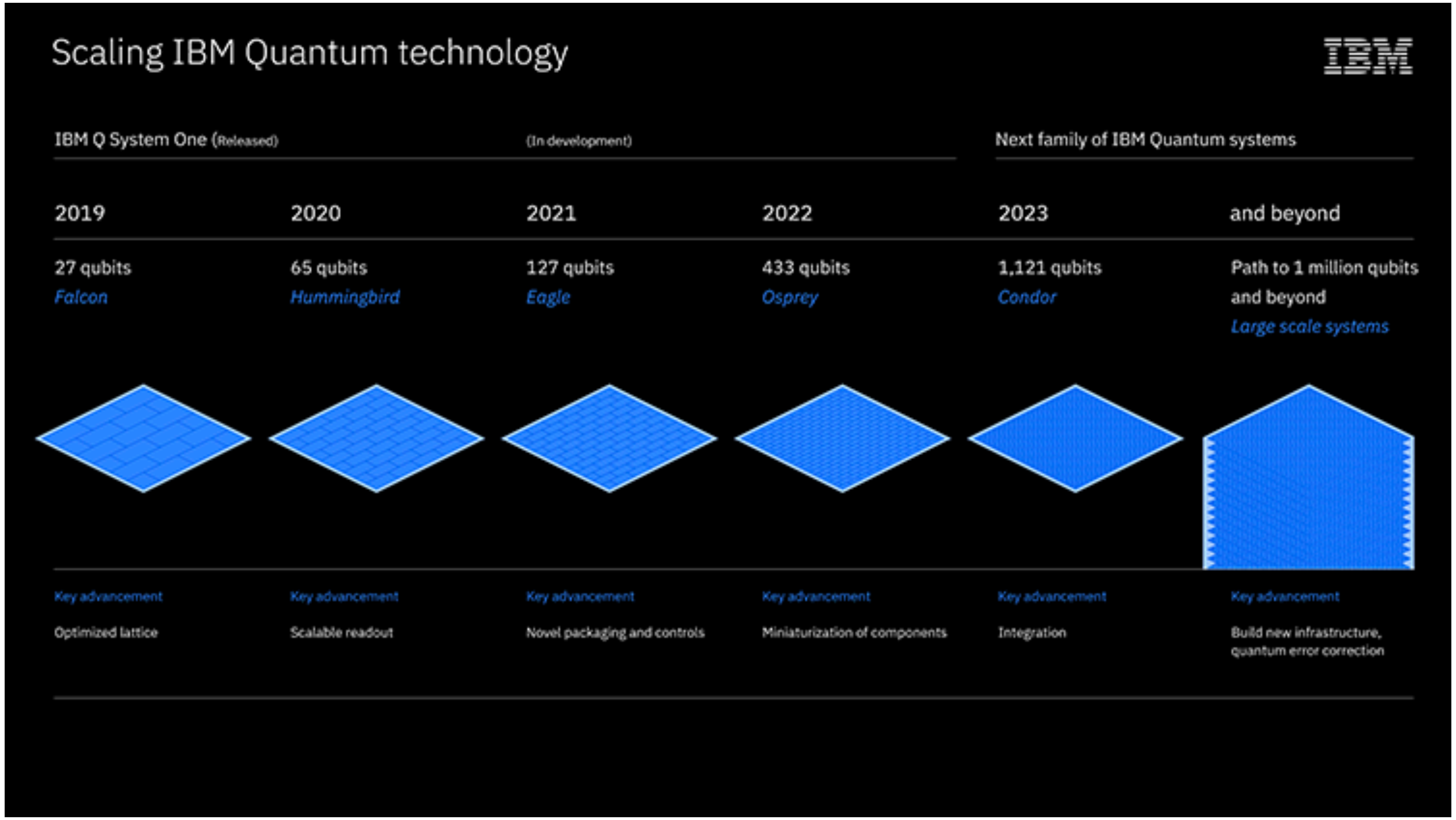


Log scale

How Many Qubits

Linear scale

Quantum Computers Today



PS: “and beyond” might be in a long time, if ever :)

Speculative Estimates…

“Predicting” quantum computers is a Bayesian game; too little information to make reliable guesses (10 scientists = 12 different predictions)

The Present and Future of Discrete Logarithm Problems on Noisy Quantum Computers

YOSHINORI AONO[1], SITONG LIU[2], TOMOKI TANAKA[3,5], SHUMPEI UNO[4,5], RODNEY VAN METER[2,5] (Senior Member, IEEE), NAOYUKI SHINOHARA[1], RYO NOJIMA[1]

scenario. Their prediction is based on their quantifier of quantum devices that they named generalized logical qubits. They predicted that a superconducting quantum device capable of solving RSA-2048 (using 4,100 qubits) would be available in the early 2050s, rather than before 2039. This is more optimistic than expert opinions [38], [39] published in 2019 and updated in 2020. Mosca and Piani say that 90% of experts predict that there is 50% or greater chance of a quantum device that can break RSA-2048 in 24 hours being released in the next 20 years.

When it Looks too Good to be True..

Factoring 2 048 RSA integers in 177 days with 13 436 qubits and a multimode memory

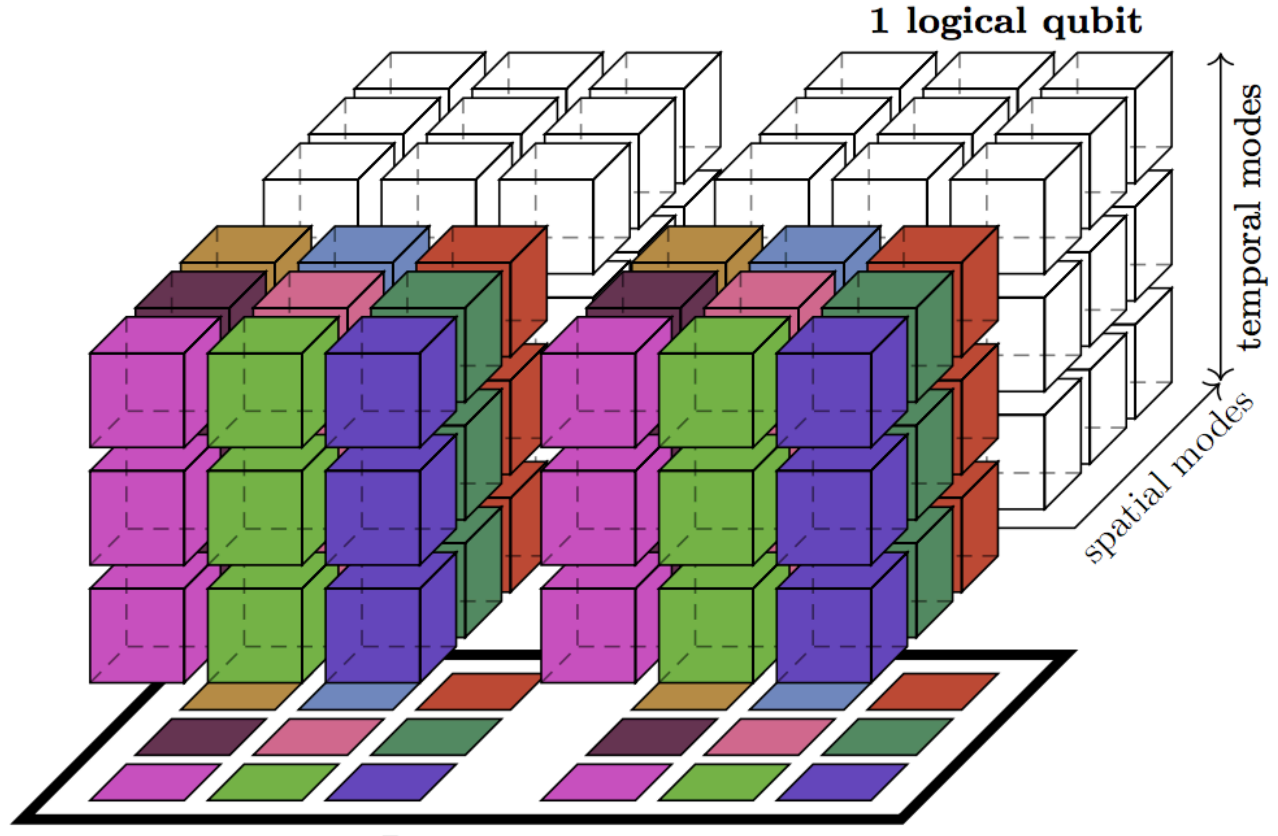
Élie Gouzien* and Nicolas Sangouard†
Université Paris–Saclay, CEA, CNRS, Institut de physique théorique, 91 191 Gif-sur-Yvette, France
(Dated: March 11, 2021)

We analyze the performance of a quantum computer architecture combining a small processor and a storage unit. By focusing on integer factorization, we show a reduction by several orders of magnitude of the number of processing qubits compared to a standard architecture using a planar grid of qubits with nearest-neighbor connectivity. This is achieved by taking benefit of a temporally and spatially multiplexed memory to store the qubit states between processing steps. Concretely, for a characteristic physical gate error rate of 10^{-3}, a processor cycle time of 1 microsecond, factoring a 2 048 bits RSA integer is shown possible in 177 days with a processor made with 13 436 physical qubits and a multimode memory with 2 hours storage time. By inserting additional error-correction steps, storage times of 1 second are shown to be sufficient at the cost of increasing the runtime by about 23 %. Shorter runtimes (and storage times) are achievable by increasing the number of qubits in the processing unit. We suggest realizing such an architecture using a microwave interface between a processor made with superconducting qubits and a multiplexed memory using the principle of photon echo in solids doped with rare-earth ions.

quant-ph] 10 Mar 2021

Introduction — Superconducting qubits form the building blocks of one of the most advanced platforms for realizing quantum computers [1]. The standard architecture consists in laying superconducting qubits in a 2D grid and making the computation using only neighboring interactions. Recent estimations showed however that fault-tolerant realizations of various quantum algorithms with this architecture would require millions physical qubits [2–4]. These performance analyses naturally raise the question of an architecture better exploiting the potential of superconducting qubits.

In developing a quantum computer architecture we



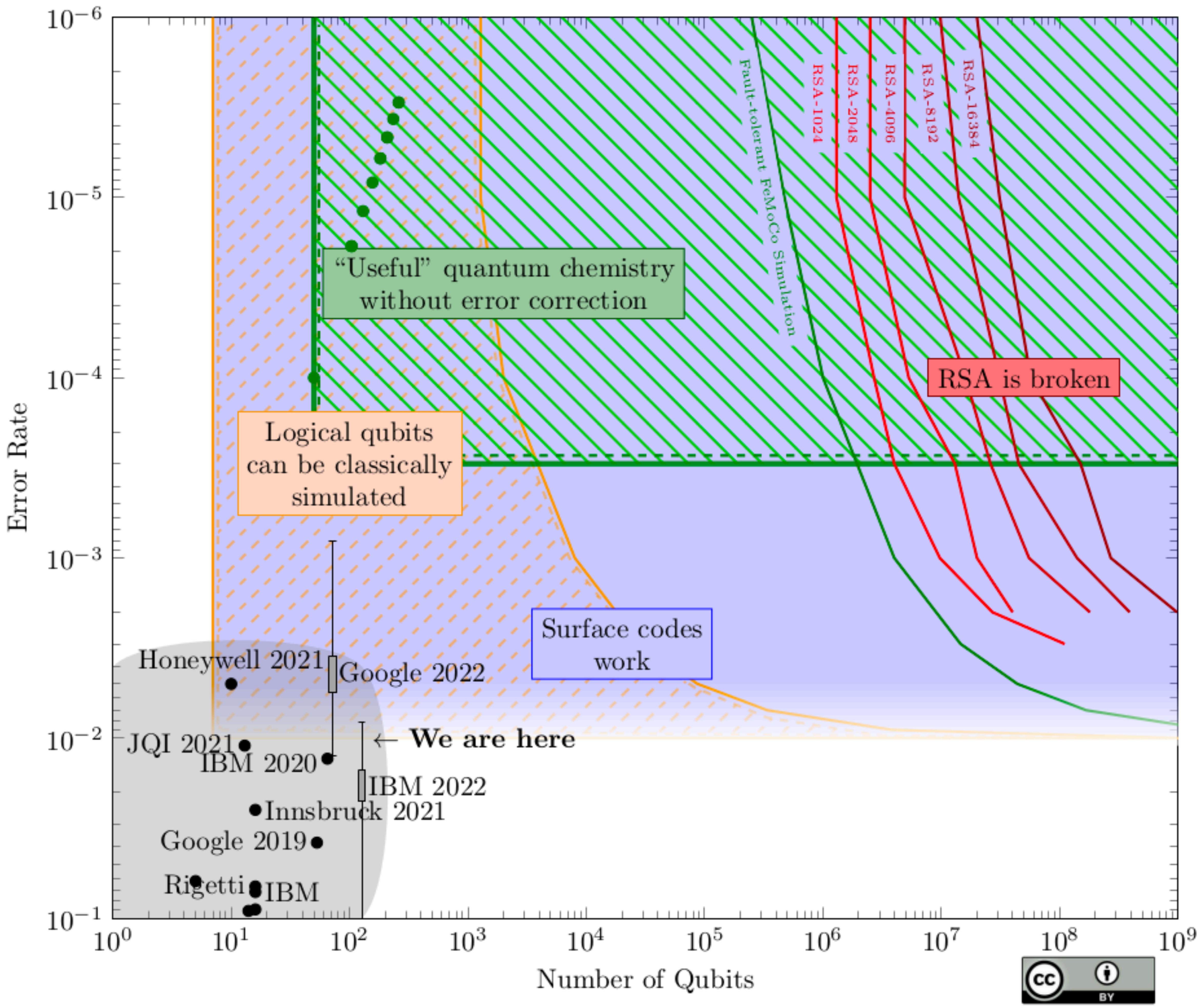
Sam Jaques
@sejaques

Replying to @veorq

Very important caveat: it needs 430 million "memory qubits"

Craig Gidney @CraigGidney · Mar 15
Replying to @quantumVerd @KikeSolanoPhys and 4 others
The paper uses a cost model where quantum memory is comparatively cheap. I'd have included the mem qubit count in the title (at n=2048 there's 13K compute qubits and 430M mem qubits) but don't see anything wrong with considering a world where mem ends up cheaper than cpu.

Quantum computing 2022 landscape



By Samuel Jacques http://sam-jaques.appspot.com/quantum_landscape_2022

Quantum Search

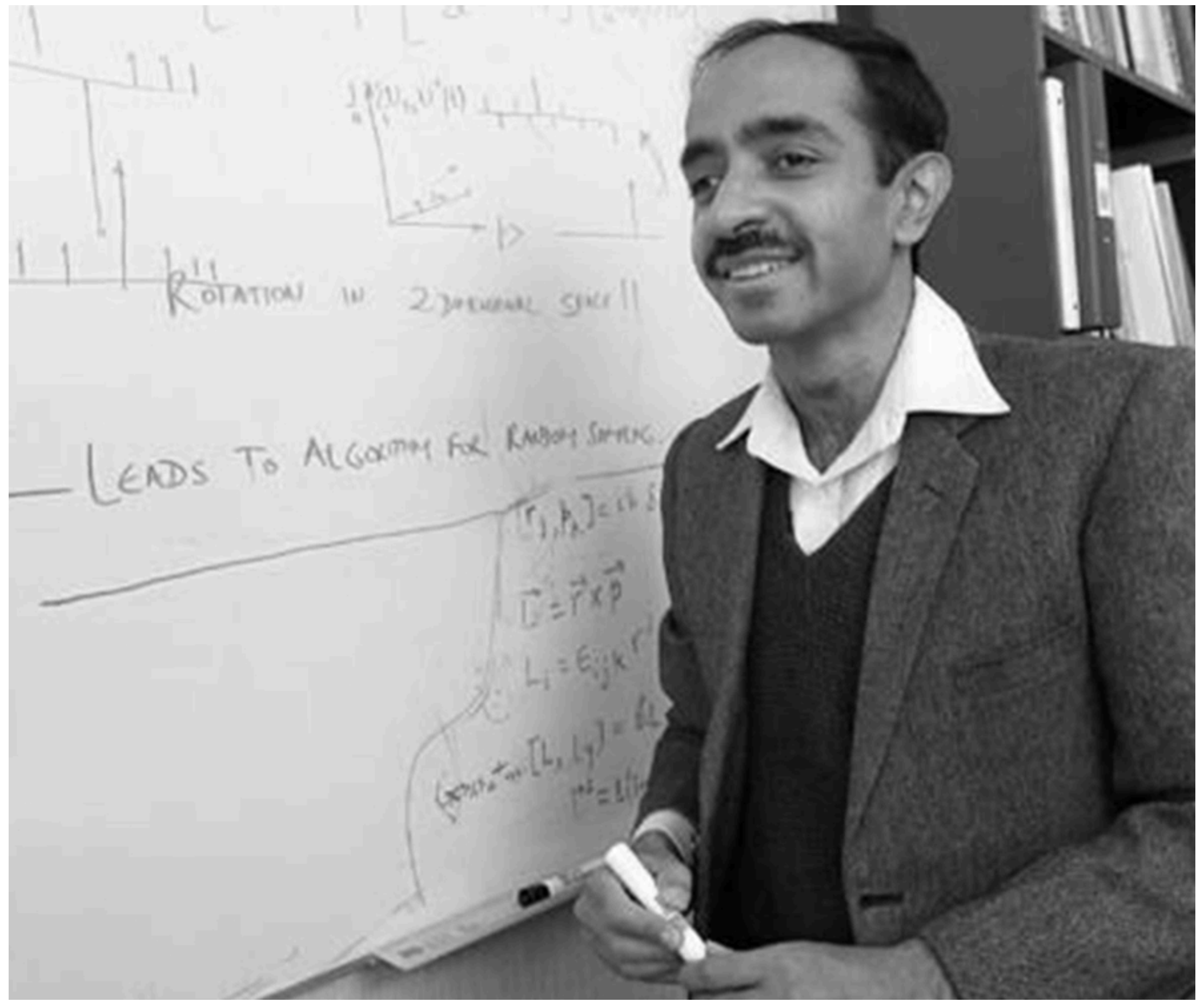
Grover's algorithm (1996)

Searches in N items in $\sqrt{N}$ queries!

AES-128 broken in $\sqrt{(2^{128})} = 2^{64}$ operations?

Caveats behind this simplistic view:

- Constant factor in **O($\sqrt{N}$)** may be huge
- Doesn't easily parallelise, as classical search does



Quantum-Searching AES Keys

	#gates		depth		#qubits
k	T	Clifford	T	overall	
128	$1.19 \cdot 2^{86}$	$1.55 \cdot 2^{86}$	$1.06 \cdot 2^{80}$	$1.16 \cdot 2^{81}$	2,953
192	$1.81 \cdot 2^{118}$	$1.17 \cdot 2^{119}$	$1.21 \cdot 2^{112}$	$1.33 \cdot 2^{113}$	4,449
256	$1.41 \cdot 2^{151}$	$1.83 \cdot 2^{151}$	$1.44 \cdot 2^{144}$	$1.57 \cdot 2^{145}$	6,681

Table 5. Quantum resource estimates for Grover's algorithm to attack AES-k, where $k \in \{128, 192, 256\}$.

https://arxiv.org/pdf/1512.04965v1.pdf

If gates are the size of a hydrogen atom (12pm) this depth is the **diameter of the solar system** (~10^{13}m), yet less than 5 grams

No doubt more efficient circuits will be designed…

Quantum-Searching AES Keys

From February 2020, better circuits found

Implementing Grover oracles for quantum key search on AES and LowMC

Samuel Jaques[1*†], Michael Naehrig[2], Martin Roetteler[3], and Fernando Virdia[4†‡]

scheme	r	#Clifford	$\#T$	$\#M$	T-depth	full depth	width	G-cost	DW-cost	p_s
AES-128	1	$1.13 \cdot 2^{82}$	$1.32 \cdot 2^{79}$	$1.32 \cdot 2^{77}$	$1.48 \cdot 2^{70}$	$1.08 \cdot 2^{75}$	1665	$1.33 \cdot 2^{82}$	$1.76 \cdot 2^{85}$	$1/e$
AES-128	2	$1.13 \cdot 2^{83}$	$1.32 \cdot 2^{80}$	$1.32 \cdot 2^{78}$	$1.48 \cdot 2^{70}$	$1.08 \cdot 2^{75}$	3329	$1.34 \cdot 2^{83}$	$1.75 \cdot 2^{86}$	1
AES-192	2	$1.27 \cdot 2^{115}$	$1.47 \cdot 2^{112}$	$1.47 \cdot 2^{110}$	$1.47 \cdot 2^{102}$	$1.14 \cdot 2^{107}$	3969	$1.50 \cdot 2^{115}$	$1.11 \cdot 2^{119}$	1
AES-256	2	$1.56 \cdot 2^{147}$	$1.81 \cdot 2^{144}$	$1.81 \cdot 2^{142}$	$1.55 \cdot 2^{134}$	$1.29 \cdot 2^{139}$	4609	$1.84 \cdot 2^{147}$	$1.45 \cdot 2^{151}$	$1/e$
AES-256	3	$1.17 \cdot 2^{148}$	$1.36 \cdot 2^{145}$	$1.36 \cdot 2^{143}$	$1.55 \cdot 2^{134}$	$1.28 \cdot 2^{139}$	6913	$1.38 \cdot 2^{148}$	$1.08 \cdot 2^{152}$	1

Eliminating the Problem: 256-bit Keys

Defeating Quantum Algorithms

A.k.a. “quantum-safe”, “quantum-resilient”

- Must not rely on factoring or discrete log problems
- Must be well-understood with respect to quantum

Why Bother?

Insurance against QC threat:

- “QC has a probability *p* work in year X and the impact would be $N for us”
- “I’d like to eliminate this risk and I’m ready to spend $M for it”

Supposedly the motivation of USG/NSA:

"*we anticipate a need to shift to quantum-resistant cryptography in the near future.”* — NSA in CNSS advisory 02-2015

NSA's Take (Aug 2021)

Q: Is NSA worried about the threat posed by a potential quantum computer because a CRQC exists?
A: NSA does not know when or even if a quantum computer of sufficient size and power to exploit public key cryptography (a CRQC) will exist.

Q: Why does NSA care about quantum computing today? Isn't quantum computing a long way off?
A: The cryptographic systems that NSA produces, certifies, and supports often have very long lifecycles. NSA has to produce requirements today for systems that will be used for many decades in the future, and data protected by these systems will still require cryptographic protection for decades after these solutions are replaced. There is growing research in the area of quantum computing, and global interest in its pursuit have provoked NSA to ensure the enduring protection of NSS by encouraging the development of post-quantum cryptographic standards and planning for an eventual transition.

Q: What are the timeframes in NSS for deployment of new algorithms, use of equipment, and national security information intelligence value?
A: New cryptography can take 20 years or more to be fully deployed to all National Security Systems. NSS equipment is often used for decades after deployment. National security information intelligence value varies depending on classification, sensitivity, and subject, but it can require protection for many decades.

https://media.defense.gov/2021/Aug/04/2002821837/-1/-1/1/Quantum_FAQs_20210804.pdf

ANSSI's Take (Apr 2022)

Avis scientifique et technique de l'ANSSI sur la migration vers la cryptographie post-quantique

14/04/2022

Dans cet avis scientifique et technique, l'ANSSI résume les différents aspects et enjeux de la menace quantique sur les systèmes cryptographiques actuels. Après un bref aperçu du contexte de cette menace, ce document introduit un planning prévisionnel de migration vers une cryptographie post-quantique, i.e. résistante aux attaques que l'émergence d'ordinateurs quantiques de grande taille rendrait possibles. L'objectif est de se prémunir par anticipation contre cette menace tout en évitant toute régression de la résistance aux attaques réalisables au moyen des ordinateurs classiques actuels. Cet avis vise à fournir une orientation aux industriels développant des produits de sécurité et à décrire les impacts de cette migration sur l'obtention des visas de sécurité délivrés par l'ANSSI [4].

Qu'est-ce qu'un ordinateur quantique ?
Les ordinateurs quantiques sont des calculateurs reposant sur des principes physiques fondamentalement différents des ordinateurs classiques actuels. Si de tels ordinateurs de grande taille sont un jour construits, ils pourraient effectuer certaines tâches beaucoup plus rapidement que ces derniers.

https://www.ssi.gouv.fr/publication/migration-vers-la-cryptographie-post-quantique/

The NIST Competition

CSRC HOME > GROUPS > CT > POST-QUANTUM CRYPTOGRAPHY PROJECT

POST-QUANTUM CRYPTO PROJECT

NEWS -- August 2, 2016: The National Institute of Standards and Technology (NIST) is requesting comments on a new process to solicit, evaluate, and standardize one or more quantum-resistant public-key cryptographic algorithms. Please see the Post-Quantum Cryptography Standardization menu at left.

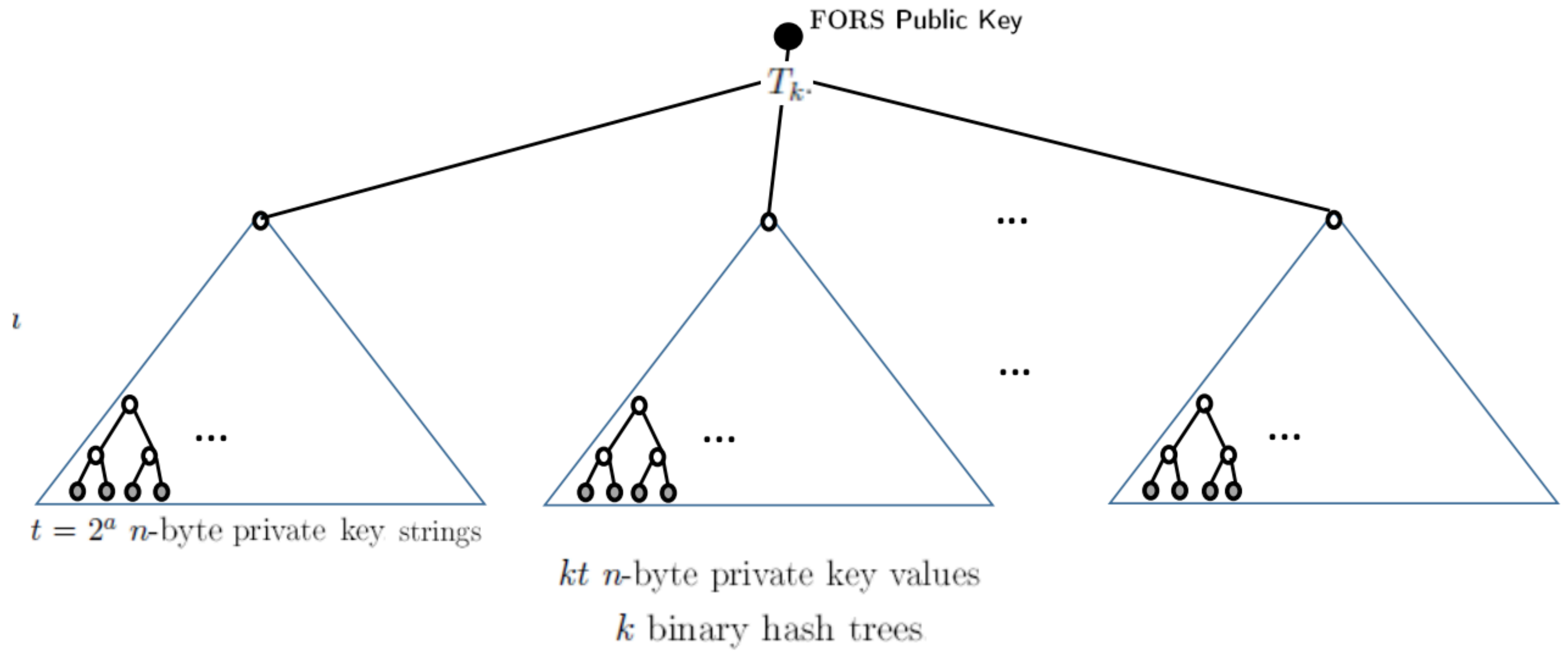
Fall 2016	Formal Call for Proposals
Nov 2017	Deadline for submissions
Early 2018	Workshop - Submitter's Presentations
3-5 years	Analysis Phase - NIST will report findings *1-2 workshops during this phase*
2 years later	Draft Standards ready

NIST Standards and Round 4

Standards announced in 2022:

- Encryption/KEM: **Kyber**
- Signature: **Dilithium**, **Falcon**, **SPHINCS+**

All *latticed-based* except SPHINCS+



Round 4 ongoing, only for encryption/KEM, all *code-based*:

BIKE, Classic McEliece, HQC

Final winners maybe in fall 2023

The Five Families

- Based on **coding theory** (McEliece, Niederreiter) – *encryption only*
  - Solid foundations from the late 1970s, large keysy
- Based on **multivariate polynomials** evaluation – *mostly signatures*
  - Based on multivariate equations’ hardness
- Based on **hash functions** and **trees** – *signatures only*
  - As secure as the hash functions, large keys and signatures
- Based on **elliptic curve isogenies**
  - More recent problem, relatively slow, some have been broken
- Based on **lattice problems…**

Lattice-Based Crypto: Intuition

Based on problems such as **learning with errors** (LWE):

S a secret vector of numbers

The attacker receives pairs of vectors (**A**, **B**)

- **A** = (A_0, …, A_{n-1}) is a vector of uniformly random numbers
- **B** = <**S**, **A**> + **E** , a vector of B_i = S_i*A_i + E_i
- **E** = (E_0, …, E_{n-1}) is an **unknown** vector or *normal*-random numbers

Attacker's goal: find **S** given many pairs (**A**, **B**)

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Attacker's goal: find **S** given many pairs (**A**, **B**)

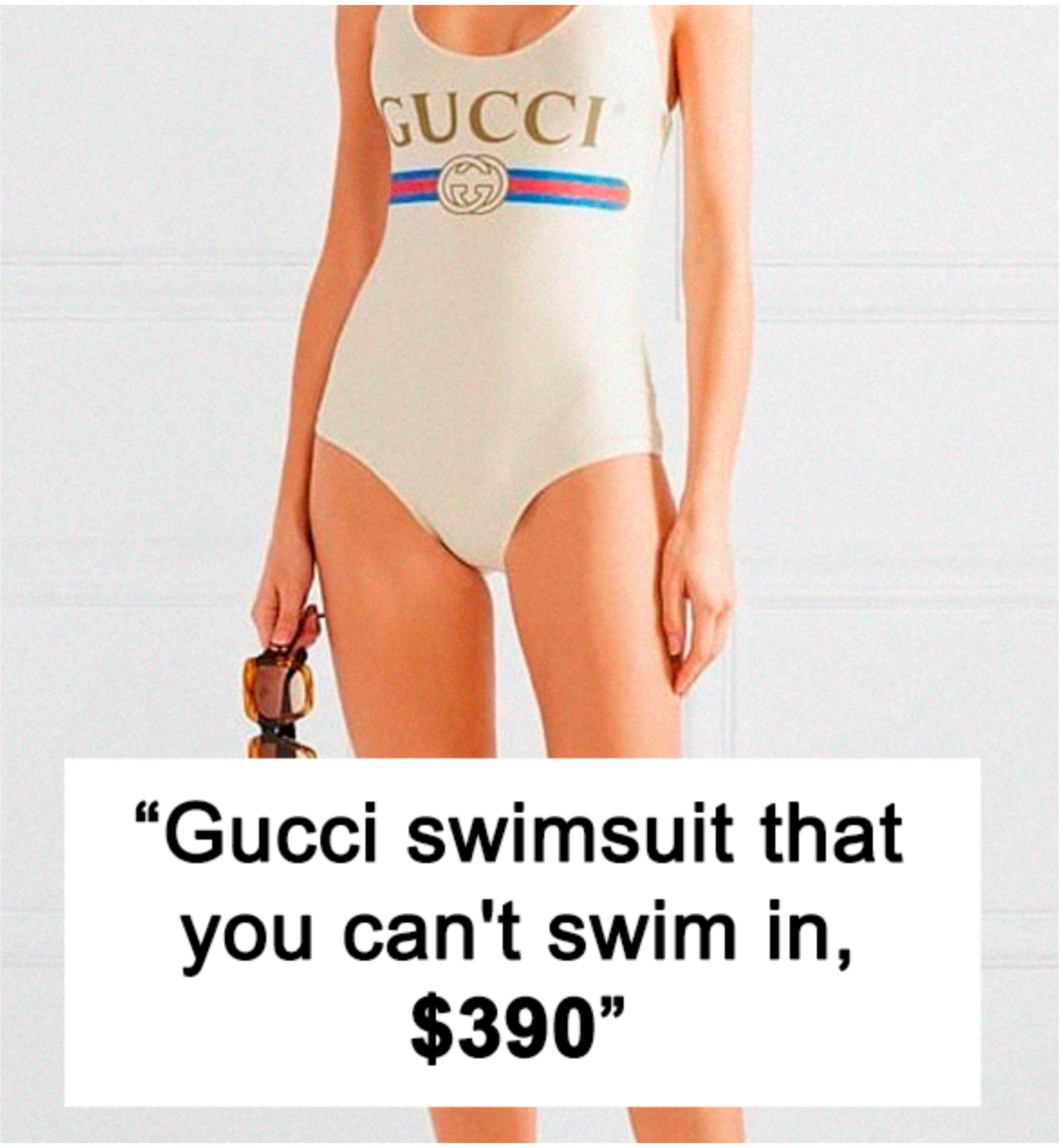
Without the errors **E**: trivial to solve (just a linear systems of equations)

With the errors **E**: **NP-hard**

Hash-Based Crypto: Intuition

“One-time signatures”, Lamport, **1979**:

1. Generate a key pair
   - Pick random strings **K_0** and **K_1** (your **private key**)
   - The public key is the two values **H**(**K_0**), **H**(**K_1**)
2. To sign the bit 0, show **K_0**, to sign 1 show **K_1**



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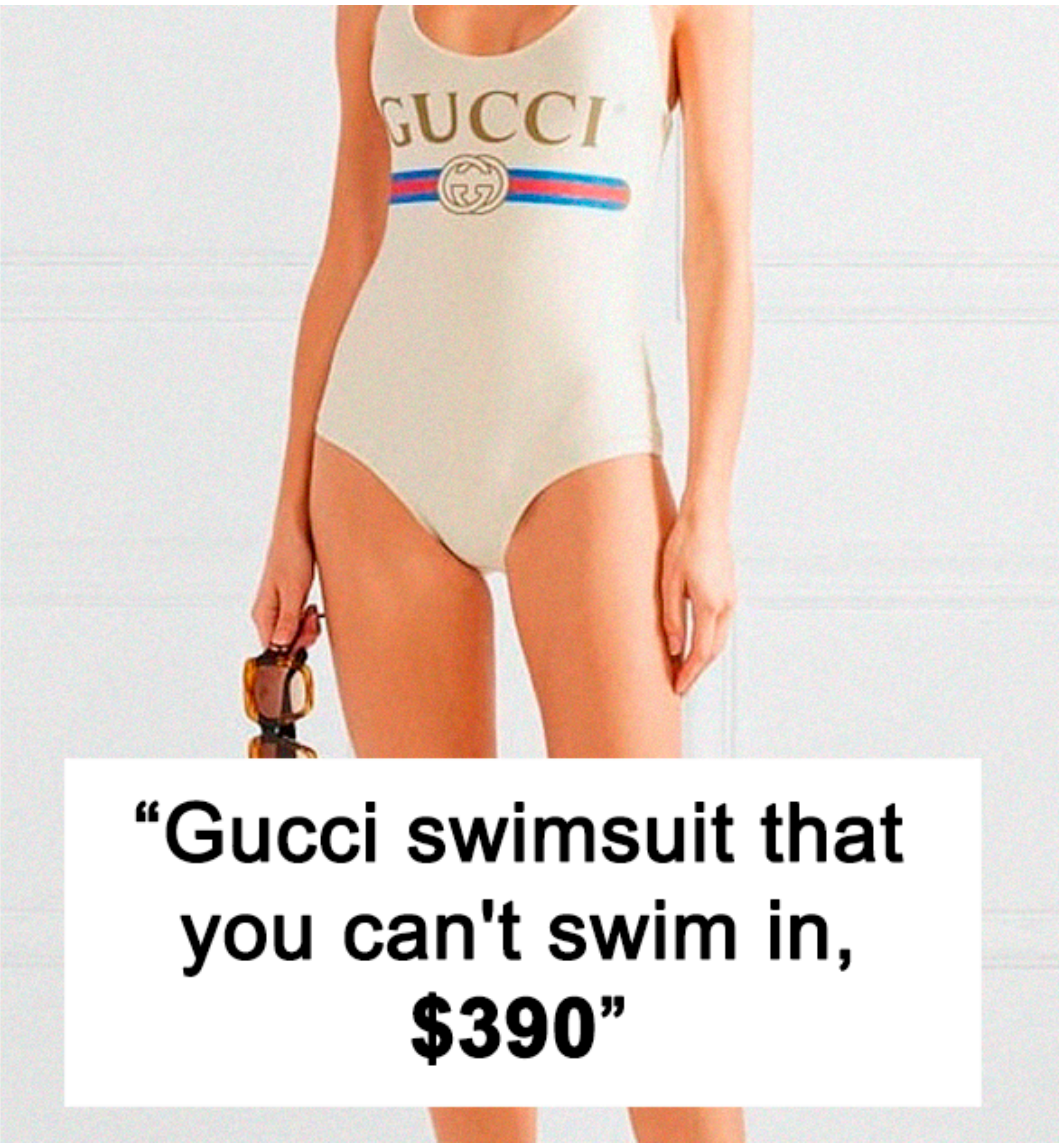
   - The public key is the two values $H(K_0)$, $H(K_1)$
2. To sign the bit 0, show K_0, to sign 1 show K_1

Problems

- Need as many keys as there are bits
- A key can only be used once
- Solution: more hashing, and trees!



it’s the most useless
and
the most expensive

Hash Crypto: Sign More than 0 and 1

Winternitz, **1979**:

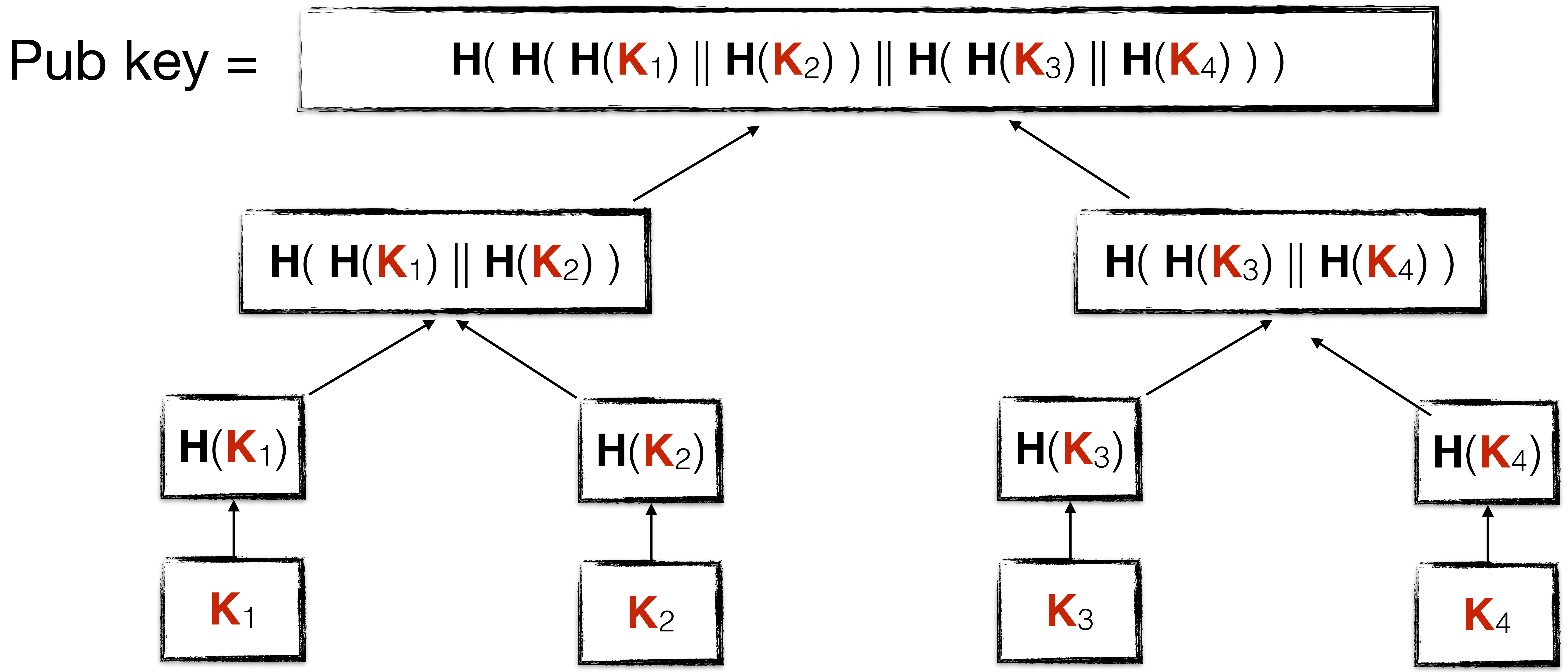
1. Public key is **H**(**H**(**H**(**H**(…. (**K**)…)) = $\mathbf{H}^{w}$ (**K**) ; that is, hash **w** times
2. To sign a number **x** in [0 .. **w** – 1], compute **S** = $\mathbf{H}^{x}$ (**K**) ; that is, hash **x** times

To verify, check that $\mathbf{H}^{w-x}$ (**S**) = public key

A key must still be used only once

Hash Crypto: From One-Time to Many-Time

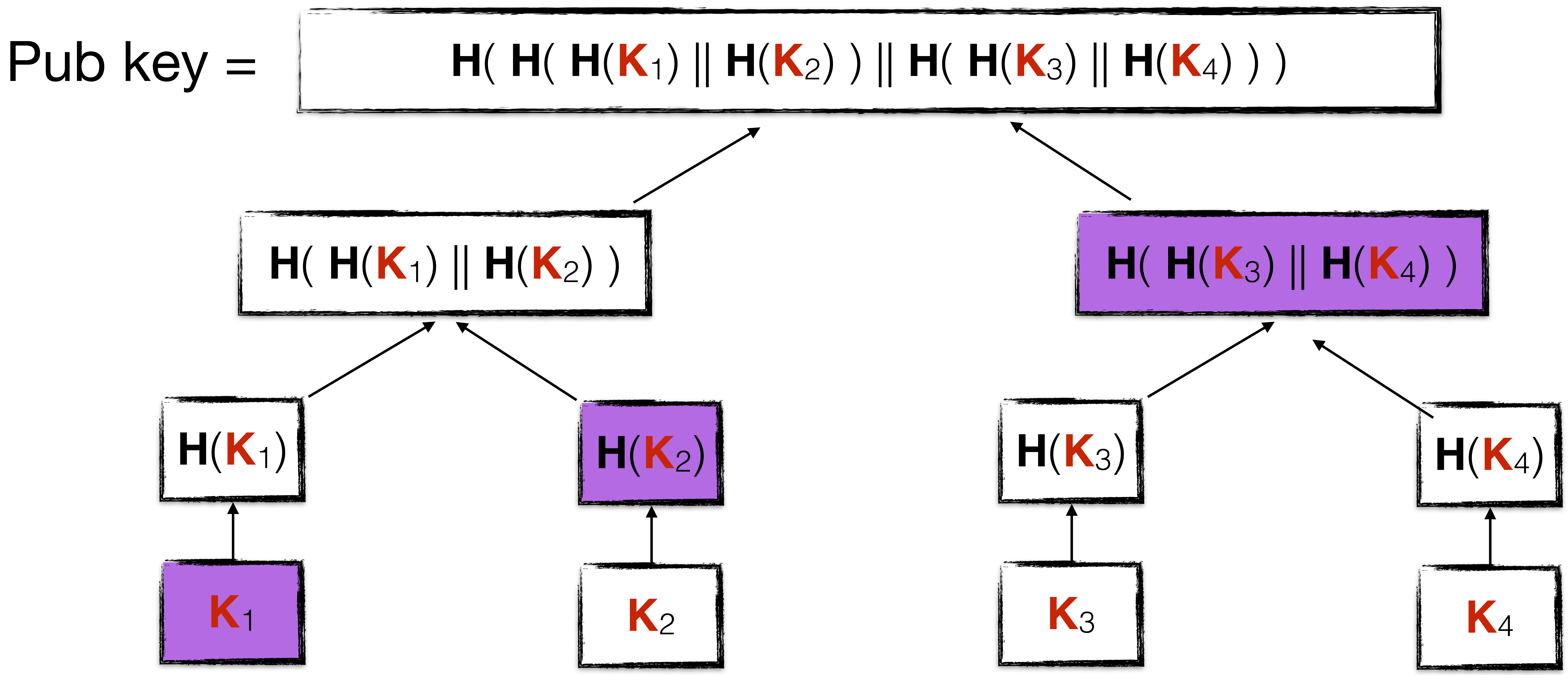
“Compress" a list of one-time keys using a **hash tree**



Hash Crypto: From One-Time to Many-Time

When a new **one-time public key** **K_i**, is used…

… give its **authentication path** to the root pub key



PQC Performance: Pretty Good!

Algorithm	Public key (bytes)	Ciphertext (bytes)	Key gen. (ms)	Encaps. (ms)	Decaps. (ms)	
ECDH NIST P-256	64	64	0.072	0.072	0.072	Elliptic curves (not post-quantum)
SIKE p434	330	346	13.763	22.120	23.734	Isogeny-based
Kyber512-90s	800	736	0.007	0.009	0.006	Lattice-based
FrodoKEM-640-AES	9,616	9,720	1.929	1.048	1.064	

Table 1: Key exchange algorithm communication size and runtime

Algorithm	Public key (bytes)	Signature (bytes)	Sign (ms)	Verify (ms)	
ECDSA NIST P-256	64	64	0.031	0.096	
Dilithium2	1,184	2,044	0.050	0.036	Lattice-based
qTESLA-P-I	14,880	2,592	1.055	0.312	
Picnic-L1-FS	33	34,036	3.429	2.584	Zero-knowledge proof-based

Table 2: Signature scheme communication size and runtime

From "Benchmarking Post-Quantum Cryptography in TLS" https://eprint.iacr.org/2019/1447

Using PQC Today

Libraries, implementations, specifications (for TLS, IPsec), standards

See https://github.com/veorq/awesome-post-quantum

open-quantum-safe / liboqs

Code | Issues 19 | Pull requests 4 | Actions | Projects 0 | W

C library for quantum-safe cryptography. https://openquantumsafe.org/

AWS Security Blog

Post-quantum TLS now supported in AWS KMS

by Andrew Hopkins | on 04 NOV 2019 | in Advanced (300), AWS Key Management Service, Security, Identity, & Compliance | Permalink | Comments | Share

PQClean / PQClean

Code | Issues 19 | Pull requests 3 | Actions | Projects 0

Clean, portable, tested implementations of post-quantum cryptography

mupq / pqm4

Code | Issues 3 | Pull requests 0 | Actions

Post-quantum crypto library for the ARM Cortex-M4

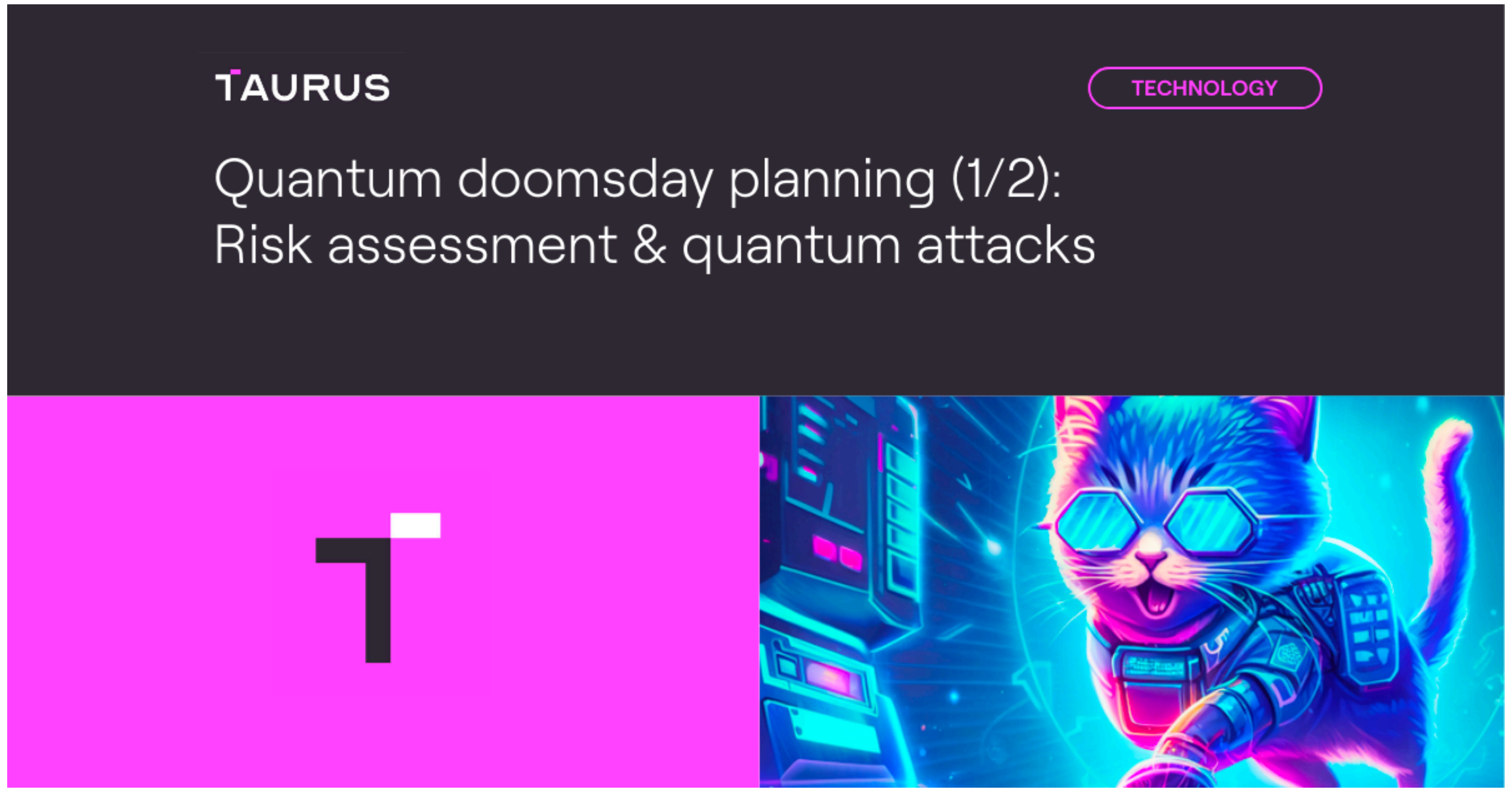
pqshield.com

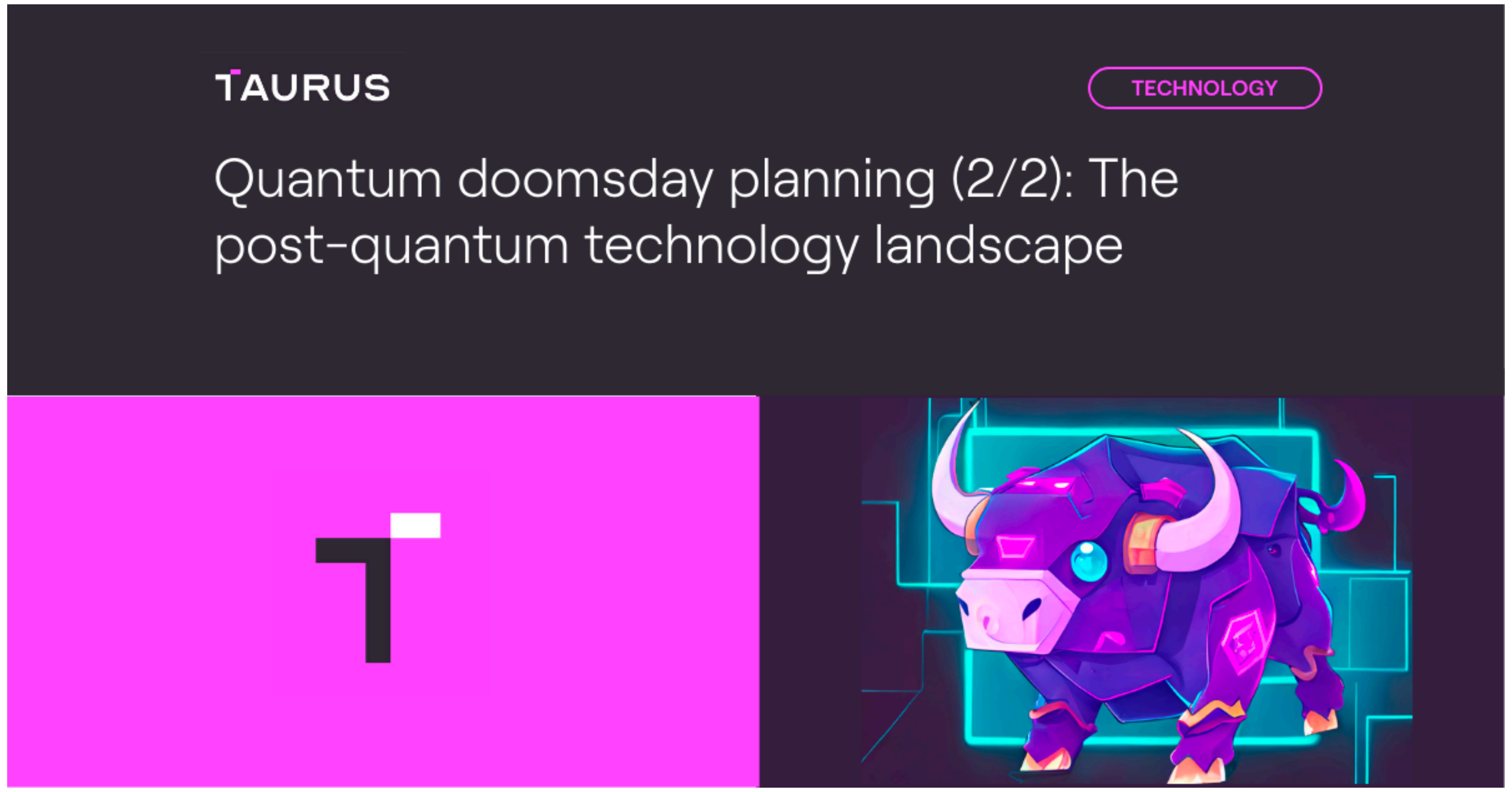
PQSHIELD

More About (Post-Quantum)

- Quantum attacks requirements for TLS, WireGuard, VPNs, Signal, 4G/5G
- Quantum computing R&D state of the art
- Cloud companies post-quantum offering

See May 2023 articles on https://blog.taurushq.com/





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Thank you

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