

### "Random numbers are absolutely essential for a crypto library, if they suck we don't even have to

for a crypto library, if they suck we don't even have to get started with encryption or anything else, because it all collapses to something trivially deterministic and therefore predictable."

Martin Boßlet



# Are these bits random? 0100110111010101010

# 

# "10 is random" makes no sense

(without a context)

# Talk instead of **random variables** with a given *distribution*

## An object may have been (pseudo)randomly generated

(we talk, a posteriori, of (pseudo)random bits)

"Randomness means different things in various fields. Commonly, it means lack of pattern or predictability in events."

Wikipedia

#### 01001101110101101010

#### 01001101110101101010

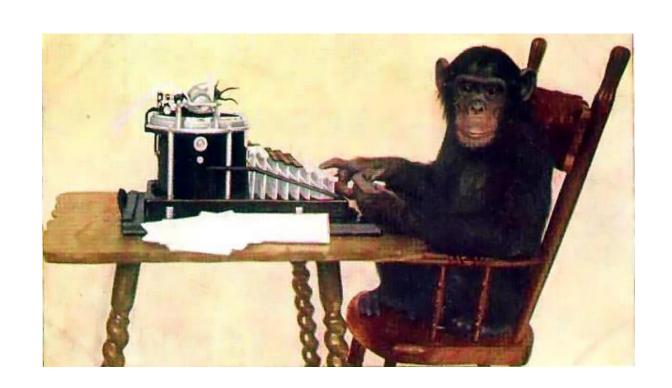
Probability =  $2^{-20}$ 

#### 000000000000000000

#### 

Probability = 2<sup>-20</sup>

### Don't be "fooled by patterns"



### PRNG are not RNGs

"Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin."

John von Neumann

### RNGs produce random bits

- non-deterministically
- thanks to analog sources
- with a deterministic postprocessing

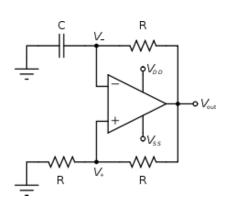
### PRNGS produce pseudorandom bits

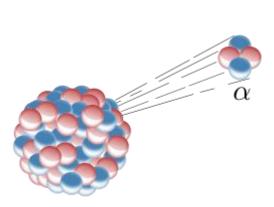
- deterministically
- from a digital seed (taken from an RNG)

Both produce "unpredictable" bits from "uncertainty"

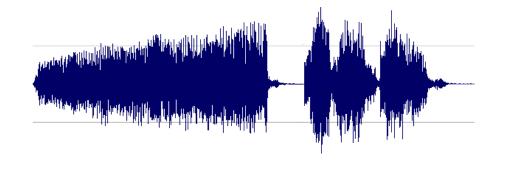
# Uncertainty comes from the real (analog/physical) world











### Uncertainty quantified with the notion of **entropy**

#### Defined for a random variable, for example

- Symmetric keys (should have as much entropy as bits)
- Public keys (as much entropy as log<sub>2</sub> #choices)

#### log<sub>2</sub> #choices

- = minimal entropy required for secure generation
- = minimal size of the RNG internal state

"I group random with stochastic or chancy, taking a random process to be one which does not operate wholly capriciously or haphazardly but in accord with stochastic or probabilistic laws."

John Earman A Primer on Determinism, 1986

Any distinction, given physics' laws?



### **Key generation** (symmetric and asymmetric)

### Challenge-response authentication protocols

#### Semantically secure encryption

IVs, nonces in block and stream ciphers Padding in RSA-OAEP, El Gamal, etc.

### Probabilistic signatures

DSA, ECDSA, etc.

### Key agreement

authenticated Diffie-Hellman, MQV, etc.

#### Side-channel defenses

Blinding, masking, jitter, etc.

### Etc. etc.

#### Where does randomness come from?

"Entropy" + postprocessing to eliminate biases (bits sampled from analog sources are often not uniformly distributed, i.e; entropy of <1 per bit)

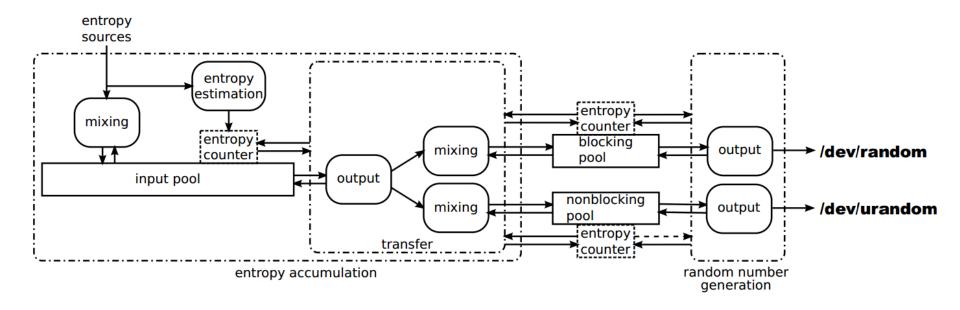
#### Implemented in OS' as

- /dev/random and /dev/urandom on unices
- CryptGenRandom on Windows
- Ad hoc tricks...

#### /dev/urandom and /dev/random

- Device file probing analog sources to gather entropy and generate random bytes
- Implemented differently on different OS'
  - Linux
  - FreeBSD
  - OpenBSD
  - etc.

```
#include <sys/types.h>
#include <sys/stat.h>
#include <fcntl.h>
#include <unistd.h>
#include <stdio.h>
int main() {
  int randint:
  int fd = open("/dev/urandom", O RDONLY);
  if (fd != -1) {
    read(fd, &randint, sizeof randint);
  printf("%08x\n", randint);
  close (fd);
  return 0;
```



- Entropy from keyboard/mouse/interrupts/disk
- 4kB entropy pool, internal mixing (linear)
- Postprocessing based on SHA-1

- Current entropy /proc/sys/kernel/random/
- Entropy appears to decrease (!) on inactivity

```
$ cat /proc/sys/kernel/random/entropy_avail
3459
$ dd if=/dev/random bs=1024 count=1 2>/dev/null | od -t x1 -An
e4 f0 78 d7 21 21 ed b2 39 e1 1e ec 70 b5 a7 77
52 bd d6 04 85 c9 0c 48 78 d3 b0 71 ef 1c a1 f6
c6 f2 dc 58 4b 76 cf 1f 61 97 ba 50 26 58 5b ad
5f fa 95 21 df 53 85 26 a0 90 ce f6 af 08 cd b2
df 4b bf 3e c9 f7 99 10 55 e2 ec e4 32 c7 88 08
09 73 8f d1 80 d2 f7 1e 3e db f1 a2 64 15 ea d0
d1 7b 50 45 64 18 71 88 12 24 5d f4 1a ee 94 70
7d 34 31 29 8a cb 2f a3 2e 7a b7 d6 89 76 3a b3
$ cat /proc/sys/kernel/random/entropy_avail
2216
$ cat /proc/sys/kernel/random/entropy_avail
2112
$ cat /proc/sys/kernel/random/entropy_avail
2005
```

- /dev/random blocks when insufficient entropy
- This is why gpg --gen-key can complain

```
$ cat /proc/sys/kernel/random/entropy_avail
644
$ dd if=/dev/random bs=1024 count=1 2>/dev/null | od -t x1 -An
    4f cd fc f9 45 63 44 db 0e b8 02 e4 a6 2a 93 ef
    68 73 a1 cf cd 7c 43 87 9f ee 4c 52 60 77 d8 59
    af fb 06 4f e9 0c 9d 67 6d cd 16 68 88 f6 c0 01
    ef 96 13 25
$ cat /proc/sys/kernel/random/entropy_avail
29
$ dd if=/dev/random bs=1024 count=1 2>/dev/null | od -t x1 -An
    d7 ec 27 75 61 17 81 02
$ ^C
```

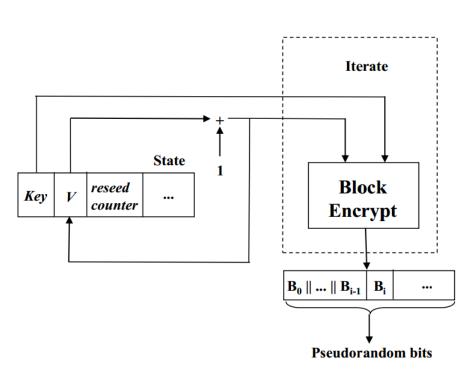
Attempting to dump 1KB blocks from /dev/random

- /dev/urandom is fine in most cases
- Most libs default to /dev/urandom for usability

openssl-1.0.1/crypto/rand/rand\_unix.c

#### CryptGenRandom

- Based on AES-CTR on Vista/7 (NIST SP 900-90)
- Less straightforward use than /dev/random need to create/release a "cryptographic context"
- Harvesting entropy from
  - Environment variables
  - Process / thread IDs
  - Time and date
  - CPU counters
  - etc.



#### CryptGenRandom

```
#include <iostream>
#include <windows.h>
#pragma comment(lib, "advapi32.lib")
int main()
  HCRYPTPROV hProvider = 0;
  if (!::CryptAcquireContextW(&hProvider, 0, 0, PROV RSA FULL, \
                               CRYPT VERIFYCONTEXT | CRYPT SILENT))
    return 1;
  const DWORD dwLength = 8;
  BYTE pbBuffer[dwLength] = {};
  if (!::CryptGenRandom(hProvider, dwLength, pbBuffer))
    :: CryptReleaseContext(hProvider, 0);
    return 1;
  for (DWORD i = 0; i < dwLength; ++i)</pre>
    std::cout << std::hex << static cast<unsigned int>(pbBuffer[i]) << std::endl;
  if (!::CryptReleaseContext(hProvider, 0))
    return 1;
```

#### When none is available?

- 1. Collect entropy from the most sources:
  - environment (e.g. env, ps aux), time, etc.
  - CPU (RDTSC, RDPMC, temperature, etc.)
  - logs (dmesg, access log, etc.)
- 2. Hash the data collected with (say) SHA-2
- 3. Seed a strong PRNG with the result

# Be. very. careful.

```
global variable seed;
RNG CreateContext()
   (seconds, microseconds) = time of day; /* Time elapsed since 1970 */
   pid = process ID; ppid = parent process ID;
   a = mklcpr(microseconds);
   b = mklcpr(pid + seconds + (ppid << 12));
    seed = MD5(a, b);
mklcpr(x) /* not cryptographically significant; shown for completeness */
   return ((0 \times DEECE66D * x + 0 \times 2BBB62DC) >> 1);
MD5() /* a very good standard mixing function, source omitted */
RNG GenerateRandomBytes()
   x = MD5(seed);
   seed = seed + 1;
   return x;
```

An attacker who has an account on the UNIX machine running the Netscape browser can easily discover the pid and ppid values used in RNG\_CreateContext() using the ps command (a utility that lists the process IDs of all processes on the system).

All that remains is to guess the time of day. Most popular Ethernet sniffing tools (including tepdump) record the precise time they see each packet. Using the output from such a program, the attacker can guess the time of day on the system running the Netscape browser to within a second. It is probably possible to improve this guess significantly. This recovers the seconds variable used in the seeding process. (There may be clock skew between the attacked machine and the machine running the packet sniffer, but this is easy to detect and compensate for.)

Of the variables used to generate the seed in Figure 2 (seconds, microseconds, pid, ppid), we know the values of seconds, pid, and ppid; only the value of the microseconds variable remains unknown. However, there are only one million possible values for it, resulting in only one million possible choices for the seed. We can use the algorithm in Figure 3 to generate the challenge and

http://www.cs.berkeley.edu/~daw/papers/ddj-netscape.html

Our second attack assumes the attacker does not have an account on the attacked UNIX machine, which means the pid and ppid quantities are no longer known. Nonetheless, these quantities are rather predictable, and several tricks can be used to recover them.

The unknown quantities are mixed in a way which can cancel out some of the randomness. In particular, even though the pid and ppid are 15bit quantities on most UNIX machines, the sum pid + (ppid << 12) has only 27 bits, not 30 (see Figure 2). If the value of seconds is known, a has only 20 unknown bits, and b has only 27 unknown bits. This leaves, at most, 47 bits of randomness in the secret key-a far cry from the 128-bit security claimed by the domestic U.S. version.

### What happened

RNG using only weak entropy sources

#### Lessons

- Don't only use weak entropy sources
   (IDs, timestamps, machine configuration, etc.)
- Estimate entropy
- A stronger post-processing doesn't matter
   (problem doesn't change if MD5 is replaced with SHA-2)

```
char buf[100];
fd = open("/dev/random", O_RDONLY);
n = read(fd, buf, sizeof buf);
close(fd);
RAND_add(buf, sizeof buf, n);
```

#### void RAND\_add(const void \*buf, int num, double entropy)

```
/* DO NOT REMOVE THE FOLLOWING CALL TO MD_Update()! */
MD_Update(&m,buf,j);
/* We know that line may cause programs such as
   purify and valgrind to complain about use of
   uninitialized data. The problem is not, it's
   with the caller. Removing that line will make
   sure you get really bad randomness and thereby
   other problems such as very insecure keys. */
```

Subject: Random number generator, uninitialised data and valgrind.

Date: 2006-05-01 19:14:00

When debbuging applications that make use of openssl using valgrind, it can show alot of warnings about doing a conditional jump based on an unitialised value. Those unitialised values are generated in the random number generator. It's adding an unintialised buffer to the pool.

The code in question that has the problem are the following 2 pieces of code in crypto/rand/md\_rand.c:

Because of the way valgrind works (and has to work), the place where the unitialised value is first used, and the place were the error is reported can be totaly different and it can be rather hard to find what the problem is.

. . .

What I currently see as best option is to actually comment out those 2 lines of code. But I have no idea what effect this really has on the RNG. The only effect I see is that the pool might receive less entropy. But on the other hand, I'm not even sure how much entropy some unitialised data has.

What do you people think about removing those 2 lines of code?

## 2 responses on the mailing list...

Both essentially said, "go ahead, remove the MD\_update line." The Debian maintainer did, causing RAND\_add not to add anything to the entropy pool but still update the entropy estimate. There were other MD\_update calls in the code that didn't use buf, and those remained. The only one that was a little unpredictable was one in RAND\_bytes that added the current process ID to the entropy pool on each call. That's why OpenSSH could still generate 32,767 possible SSH keys of a given type and size (one for each pid) instead of just one.

### What happened

 "Optimization" of sloppy code dramatically reduced the entropy of the PRNG

#### Lessons

- Beware of "optimizations" and of "clever code"
- Do not tweak OpenSSL (unless you really have to)
- Have the crypto code review by experts

#### Randomness for randomization

### Often to generate random objects, like

- Numbers in an arbitrary range
- RSA moduli

#### How is this done?

- How to generate a random number in {1,2,3} ?
- How to generate a random 1k RSA modulus?

This generates a string of 16 digits in {0,1,..,9}:

```
Cryptocat.random = function() {
        var x, o = '';
        while (o.length < 16) {
                x = state.getBytes(1);
                if (x[0] <= 250) {
                        o += x[0] \% 10;
        return parseFloat('0.' + o);
```

What's wrong?

getBytes is a strong PRNG, based on Salsa20

- Selects integers in  $\{0,1,...,250\} \Rightarrow 251$  possible values:
  - 25 values give a 1, 25 values a 2, ..., 25 values a 9
  - 26 values give a 0

getBytes is a strong PRNG, based on Salsa20

- => 16-digit string has entropy 45 bits instead of 53
- => Bruteforce would take on average 2<sup>44</sup> instead of 2<sup>52</sup>

### What happened

 A bias was introduced in the postprocessing of strong random bits

#### Lessons

- Distinguish PRNG from sampling algorithms
- Make sure the algorithm implements a uniformly random sampling
- Test, test, test

### Bug #4: Routers, firewalls, switches...

```
prng.seed(seed)
p = prng.generate_random_prime()
prng.add_randomness(bits)
q = prng.generate_random_prime()
N = p*q
```

What can go wrong?

### Bug #4: Routers, firewalls, switches...

```
prng.seed(seed)
p = prng.generate_random_prime()
prng.add_randomness(bits)
q = prng.generate_random_prime()
N = p*q
```

What if two devices start with a same seed? (and later gather distinct entropy bits from their activity)

### Bug #4: Routers, firewalls, switches...

Even more alarmingly, we are able to compute the private keys for 64,000 (0.50%) of the TLS hosts and 108,000 (1.06%) of the SSH hosts from our scan data alone by exploiting known weaknesses of RSA and DSA when used with insufficient randomness. In the case of RSA, distinct moduli that share exactly one prime factor will result in public keys that appear distinct but whose private keys are efficiently computable by calculating the greatest common divisor (GCD). We implemented an algorithm that can compute the GCDs of all pairs of 11 million distinct public RSA moduli in less than 2 hours

## https://factorable.net

### Bug #4: Routers, firewall, switches...

### What happened

– Entropy reuse + structured sampling => Fail

#### Lessons

- RSA key generation is not failure-friendly
- On low-entropy platforms, wait until sufficient entropy before generating crypto secrets
- Test, test, test

#### Which PRNG should we use?

C(++): OpenSSL, NaCl (TODO)

Python/Ruby/Perl/Go etc.: a crypto-strong module seeding from /dev/(u) random or CryptGenRandom

If a hardware RNG is available, use it (RdRand..)

Need to rely on a specific algorithm?

- Secure-blockcipher-CTR, secure stream cipher
- Better to implement reseeding (with FS/BS)
- Be very careful get your code reviewed, tested

```
* Return a random password. Sourced from mt rand, so it's not particularly secure.
  @todo hash random numbers to improve security, like generateToken()
* @return \string New random password
static function randomPassword() {
       global $wgMinimalPasswordLength;
       $pwchars = 'ABCDEFGHJKLMNPQRSTUVWXYZabcdefghjkmnpqrstuvwxyz';
       $l = strlen( pwchars ) - 1;
       $pwlength = max( 7, $wgMinimalPasswordLength );
        $digit = mt_rand( 0, $pwlength - 1 );
        Snp = '':
        for ( $i = 0; $i < $pwlength; $i++ ) {
                $np .= $i == $digit ? chr( mt_rand( 48, 57 ) ) : $pwchars{ mt_rand( 0, $l ) };
       return $np;
```

```
/**
  * Generate a looking random token for various uses.
  *
  * @param $salt \string Optional salt value
  * @return \string The new random token
  */
public static function generateToken( $salt = '' ) {
        $token = dechex( mt_rand() ) . dechex( mt_rand() );
        return md5( $token . $salt );
}
```

This generates password-reset tokens...

mt\_srand(seed)/mt\_rand(min, max): mt\_rand is the interface for the Mersenne Twister (MT) generator [15] in the PHP system. In order to be compatible with the 31 bit output of rand(), the LSB of the MT function is discarded. The function takes two optional arguments which map the 31 bit number to the [min, max] range. The mt\_srand() function is used to seed the MT generator with the 32 bit value seed; if no seed is provided then the seed is provided by the PHP system.

(Argyros/Kiayias, 2012)

19937-bit state, but fully linear update and 32-bit seed:

$$x_{k+n} = x_{k+m} \oplus ((x_k \land 0x80000000) | (x_{k+1} \land 0x7fffffff))A$$

$$xA = \begin{cases} (x \gg 1) & \text{if } x^{31} = 0\\ (x \gg 1) \oplus a & \text{if } x^{31} = 1 \end{cases}$$

A session identifier preimage completely determines the seed of the mt\_rand() and rand() PRNGs!

(Argyros/Kiayias, 2012)

- ⇒ Weak RNG can be exploited to
  - Hijack sessions
  - Predict temporary passwords

### What happened

Weak RNG used for security purposes

#### Lessons

- Avoid non-crypto PRNGs, even for other applications than key generation
- In PHP, better use openssl\_random\_pseudo\_bytes
   (and check the \$crypto strong flag)
- Don't use PHP's rand() or mt\_rand(); C's
  random(3), rand(3); LFSRs; etc.

Can I make sure that my entropy is OK? That my PRNG implementation is OK?

# **TESTING**

will detect many randomness bugs will NOT detect many randomness bugs

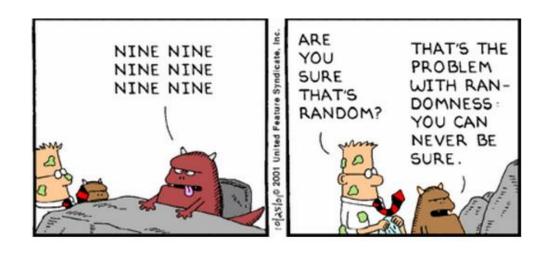
## Testing (P)RNGs

#### Test as much as possible

- The **PRNG** function
  - Determinism, through test vectors in different settings
  - Usual software bugs, memory leaks, etc.
  - Pseudorandomness statistical tests
- The underlying entropy source
  - How much bits should be expected? Does if fail securely?
  - Is the entropy quality consistent accross OSs/hardware?
- The usage of pseudorandom bits
  - Sampling algorithm for software bugs
  - Distribution of sampled objects (is it really uniform?)

### Statistical tests: background

Can provide evidence that a PRNG is weak, but NOT evidence that a PRNG is *cryptographically* strong



Based on **hypothesis testing** methods result = likelihood that the hypothesis was correct

Typically hypothesis: uniform distribution

### Statistical tests: background

Remember that

- 1. Cryptographic weaknesses will NOT be detected by statistical tests
- 2. It is easy to design a weak PRNG that will successfully pass all statistical tests

**Corollary**: backdoors in hardware (P)RNGs can remain undetectable by statistical tests and black-box query

### Statistical test suites

Test with deterministic seeds

Use large enough samples (a few megabytes)
Check the encoding in your test sample
(e.g., base-64 will look non-random if tested as binary data)

If not sure how to interpret the result, compare with a reliable source of randomness:

dd if=/dev/urandom of=buf bs=1014 count=1024

### Ent (www.fourmilab.ch/random)

#### Easiest to use, limited set of tests

```
$ ./ent onekb
Entropy = 7.756598 bits per byte.

Optimum compression would reduce the size of this 1014 byte file by 3 percent.

Chi square distribution for 1014 samples is 315.48, and randomly would exceed this value 0.59 percent of the times.

Arithmetic mean value of data bytes is 125.1844 (127.5 = random).

Monte Carlo value for Pi is 3.266272189 (error 3.97 percent).

Serial correlation coefficient is 0.023406 (totally uncorrelated = 0.0).
```

#### Results for 1 KB from /dev/urandom

### Ent (www.fourmilab.ch/random)

#### Easiest to use, limited set of tests

```
$ ./ent onemb
Entropy = 7.999806 bits per byte.

Optimum compression would reduce the size of this 1028196 byte file by 0 percent.

Chi square distribution for 1028196 samples is 277.22, and randomly would exceed this value 16.21 percent of the times.

Arithmetic mean value of data bytes is 127.4411 (127.5 = random).

Monte Carlo value for Pi is 3.135441103 (error 0.20 percent).

Serial correlation coefficient is 0.001117 (totally uncorrelated = 0.0).
```

#### Results for 1 MB from /dev/urandom

### Ent (www.fourmilab.ch/random)

#### Easiest to use, limited set of tests

```
Entropy = 7.999984 bits per byte.

Optimum compression would reduce the size of this 10383360 byte file by 0 percent.

Chi square distribution for 10383360 samples is 224.86, and randomly would exceed this value 91.33 percent of the times.

Arithmetic mean value of data bytes is 127.5012 (127.5 = random).

Monte Carlo value for Pi is 3.141561113 (error 0.00 percent).

Serial correlation coefficient is -0.000111 (totally uncorrelated = 0.0).
```

#### Results for 10 MB from /dev/urandom

### Diehard (http://www.stat.fsu.edu/pub/diehard/)

#### Suite of statistical tests, less simple to interpret

```
Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000
  chisquare equiv normal p value Results for COUNT-THE 1's in specified bytes:
                                                        .695213
                  2 to
                            2612.60
                                           1.592
                 3 to 10
                                           -.795
                  4 to 11
                                           -.116
                                           -.236
                                           -.676
                11 to
                                           .048
1.278
                 12 to
                            2590.39
                13 to 20
                                           1.030
                14 to 21
                15 to 22
                                           2.135
                16 to
                                           2.806
                                            .642
                 17 to 24
                                          -1.014
                19 to 26
                                           2.775
                 21 to 28
                 23 to 30
                24 to
```

```
Chisquare with 6 d.o.f. =
                  For a sample of size 500:
                                                 mean
                          using bits 3 to 26
                                                 2.086
duplicate
                 number
                observed
spacings
                             expected
                  56.
                            67.668
                 135.
                 137.
                               5.31 p-value=
                  For a sample of size 500:
                                                 mean
                                                 1.992
                          using bits 4 to 27
duplicate
                 number
spacings
                observed
                              expected
                  67.
                            67.668
                 139.
                           135.335
```

# p-values should be uniformly distributed

(e.g., a p-value consistently close to zero indicates a bias)

## **CONCLUSION:**

## The 10 commandments



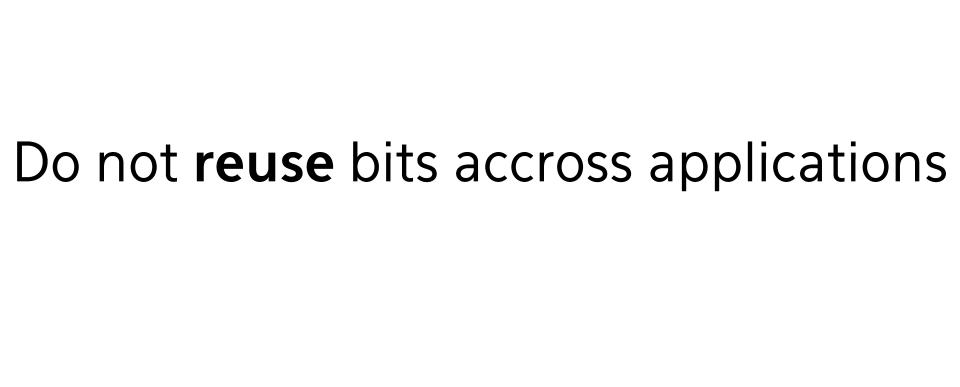
Do not rely only on **predictable entropy sources** like timestamps, PIDs, temperature sensors, etc.

## Do not rely only on non-crypto functions

Do not use **non-crypto PRNGS** like LFSRs, LCGs, Mersenne Twister, etc.

## Do not use RaaS (things like <a href="www.random.org">www.random.org</a>) -> random bits may be shared or reused

Do not design your own PRNG, even if it's based on strong crypto (unless you know what you're doing)



Do not conclude that a PRNG is secure just because it passes all the **statistical tests** (Ent, Diehard, etc.)

Do not assume that a crypto-secure PRNG does necessarily provide **forward or backward** secrecy, would the internal state leak to an attacker.

Do not directly use "entropy" as pseudorandom data (entropy from analog sources is often biased)

## Do not use random bits if you don't have to

(it's safer to use a counter as a nonce, for example)

## **TEST TEST!**

